σ Journal of Engineering and Natural Sciences
Mühendislik ve Fen Bilimleri Dergisi
Sigma
2006/1

Araştırma Makalesi / Research Article

SECOND ORDER REFINED PLASTIC HINGE ANALYSIS OF STEEL PLANE FRAMES

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Geliş/Received: 01.09.2004 Kabul/Accepted: 23.02.2006

ABSTRACT

In this paper, refined plastic hinge analysis method which accounts for material and geometric nonlinearities of the steel structures is given. For the purpose of determining the realistic steel frame behavior; gradual yielding, second order effects, and geometric imperfections are presented. The refined plastic analysis results are verified by comparison of the plastic zone analysis results by using a calibration frame. Also, three different methods of geometric imperfection modeling are investigated.
Keywords: Refined plastic hinge, second order, nonlinear analysis.

ÇELİK DÜZLEM ÇERÇEVELERİN İKİNCİ MERTEBE KADEMELİ PLASTİK MAFSAL ANALİZİ

ÖZET

Bu çalışmada, çelik yapıların malzeme ve geometri bakımından lineer olmayan davranışını dikkate alan kademeli plastik mafsal analizi yöntemi verilmiştir. Çelik çerçeve davranışının gerçekçi biçimde belirlenmesi amacıyla; malzemenin kademeli akma davranışı, ikinci mertebe etkiler ve geometrik kusurların hesaplara dahil edilmesi sunulmuştur. Kademeli plastik mafsal analizi sonuçları ile plastik zon analizi sonuçları kalibrasyon çerçevesi kullanılarak karşılaştırılmıştır. Ayrıca, geometrik kusurların modellenmesi üç ayrı yöntem ile incelenmiştir.
Anahtar Sözcükler: Kademeli plastik mafsal, ikinci mertebe, doğrusal olmayan analiz.

1. INTRODUCTION

During the past 20 years, numerous analytical models have been developed for second-order inelastic analysis of steel frames. In general these models may be categorized into two main types: plastic zone (also called distributed plasticity) and plastic hinge (also called concentrated plastic hinge) analysis. The plastic zone model follows explicitly the gradual spread of yielding throughout the volume of the structure. Plastification in the members is modeled by discretization of members into several beam-column elements and subdivision of the cross section into many fibers [1]. The effects of residual stress, geometric imperfections, and material strain hardening can all be accounted for in a plastic-zone analysis model and generally considered as an “exact” method and referred as advanced analysis [2]. However, this type of analysis is too computationally intensive for general design use, and because of its complexity and cost, it has

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not yet found application in ordinary practice [3]. Even if such analysis methods should become generally available and reliable, a more efficient procedure to assess the structural performance and failure modes of a system would be useful. Plastic hinge based methods of analysis hold the promise to fulfill these requirements.

In conventional plastic hinge based analysis, inelasticity in frame elements is assumed to concentrate at zero length plastic hinges. Regions in the frame elements other than at the plastic hinges are assumed to behave elastically [4]. If the cross section forces at any particular locations in an element are less than the cross-sectional plastic capacity, elastic behavior is assumed. If the section plastic capacity is reached, a plastic hinge is formed and the element stiffness matrix is adjusted to account for the presence of a plastic hinge. The cross sectional response after the formation of a plastic hinge is usually assumed to be perfectly plastic with no strain hardening [5]. The conventional elastic plastic analysis does not represent stiffness degradation due to distributed yielding and associated P-δ effects within the member. As a result of this problem, the elastic plastic hinge analysis method may wrongly predict the strength and stiffness of component members in a frame; therefore it can not be classified as advanced analysis in general [3]. Since, only the plastic zone analysis has been classified as an advanced analysis technique. Australian Standard AS4100 [6] and Eurocode 3 [7] are the only design specifications that explicitly allow engineers disregard member capacity checks if a plastic zone analysis is employed. This study investigates an improved plastic hinge based method called the refined plastic hinge analysis method. The refined plastic hinge analysis method is a practical advanced analysis technique for frame analysis [2, 8]. The refined plastic hinge approach adopts a suitable stiffness degradation function that presents the distributed yielding behavior of beam columns. The present wok is limited to two dimensional steel frames under static loads only and since all members assumed to be sufficiently braced such that flexural and lateral torsional buckling is not considered.
The following basic assumptions are used for modeling of a beam-column element [4, 9].

1. All elements are initially straight and prismatic.
2. Plane cross section remains plain after deformation.
3. Local buckling and lateral torsional buckling are not considered. All members are assumed to be fully compact and adequately braced.
4. Large displacements are allowed, but strains are small.
5. The element stiffness formulation is based on beam-column stability functions considering axial and bending deformations.
6. Strain hardening is not considered. Plastic hinges can not sustain additional loads.
7. Reduction of torsional and shear stiffness is not considered in plastic hinge.

2. REFINED PLASTIC HINGE ANALYSIS

The important attributes which affect the behavior of steel framed structures may be grouped into two categories: geometric and material nonlinearities. The geometric nonlinearity includes second-order effects associated with P-δ and P-Δ effects and geometric imperfections. The material nonlinearity includes gradual yielding associated with the influence of residual stresses and flexure.

In the refined plastic hinge approach, the element stiffness is assumed to degrade according to a prescribed function after the element end forces exceed a predefined initial yield function [2]. Refined plastic hinge analysis incorporates consideration of second order geometry, gradual yielding (associated with residual stress and flexure), and geometric imperfections to the analysis of steel frames.

2.1. Determining Second Order Effects by Using Stability Functions

Stability functions are used to capture second order effects, since they can account for the effect of the axial force on the bending stiffness reduction of each member and also used to minimize modeling and solution time. Generally only one or two elements are needed per a member [10].

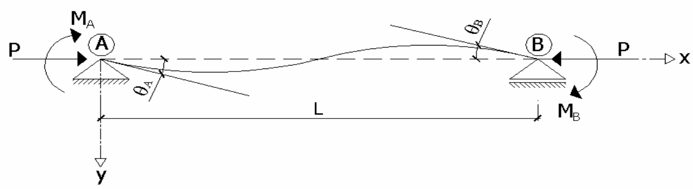

Figure 1. Beam-column element

Considering the prismatic beam-column element in Figure 1, the incremental force–displacement relationship of this element can be written as:

$$\begin{Bmatrix} \overline{M_A} \\ \overline{M_B} \\ \overline{P} \end{Bmatrix} = \frac{E\,I_i}{L_i}\begin{bmatrix} S_1 & S_2 & 0 \\ S_2 & S_1 & 0 \\ 0 & 0 & A/I \end{bmatrix}\begin{Bmatrix} \overline{\theta_A} \\ \overline{\theta_B} \\ \overline{e} \end{Bmatrix} \tag{1}$$

Where S_1 and S_2 are stability functions, $\overline{M_A}$ and $\overline{M_B}$ are incremental end moments, $\overline{P}$ is incremental axial force, $\overline{\theta_A}$ and $\overline{\theta_B}$ are incremental joint rotations, $\overline{e}$ is incremental axial displacement, A is area, I is moment of inertia, L is length of beam-column element and E is the modulus of elasticity. The stability functions given by equation (1) are written as:

$$S_1 = \frac{\pi\sqrt{\rho}\,\sin\left(\pi\sqrt{\rho}\right) - \pi^2\rho\cos\left(\pi\sqrt{\rho}\right)}{2 - 2\cos\left(\pi\sqrt{\rho}\right) - \pi\sqrt{\rho}\sin\left(\pi\sqrt{\rho}\right)} \quad (P<0) \tag{2}$$

$$S_2 = \frac{\pi^2\rho - \pi\sqrt{\rho}\,\sin\left(\pi\sqrt{\rho}\right)}{2 - 2\cos\left(\pi\sqrt{\rho}\right) - \pi\sqrt{\rho}\sin\left(\pi\sqrt{\rho}\right)} \quad (P<0) \tag{3}$$

$$S_1 = \frac{-\pi\sqrt{\rho}\sinh\left(\pi\sqrt{\rho}\right) + \pi^2\rho\cosh\left(\pi\sqrt{\rho}\right)}{2 - 2\cosh\left(\pi\sqrt{\rho}\right) + \pi\sqrt{\rho}\sinh\left(\pi\sqrt{\rho}\right)} \quad (P>0) \tag{4}$$

$$S_2 = \frac{-\pi^2\rho + \pi\sqrt{\rho}\sinh\left(\pi\sqrt{\rho}\right)}{2 - 2\cosh\left(\pi\sqrt{\rho}\right) + \pi\sqrt{\rho}\sinh\left(\pi\sqrt{\rho}\right)} \quad (P>0) \tag{5}$$

where $\rho = P/\left(\pi^2 EI/L^2\right)$, P is positive in tension.

The numerical solutions obtained from the equations (2-5) are indeterminate when the axial force is zero. To circumvent this problem and to avoid the use of different expressions for S_1 and S_2 for a different sign of axial forces, a set of expressions that make use of power-series expansions to approximate the stability functions. The power-series expressions have been shown to converge to a high degree of accuracy within the first ten terms of the series expansions [11]. Alternatively, if the axial force in the member falls within the range $-2.0 \le \rho \le 2.0$, the following simplified expressions may be used to closely approximate the stability functions [12]:

$$S_1 = 4 + \frac{2\pi^2\rho}{15} - \frac{(0.01\rho + 0.543)\rho^2}{(4+\rho)} - \frac{(0.004\rho + 0.285)\rho^2}{(8.183+\rho)} \tag{6}$$

$$S_2 = 2 - \frac{\pi^2\rho}{30} - \frac{(0.01\rho + 0.543)\rho^2}{(4+\rho)} - \frac{(0.004\rho + 0.285)\rho^2}{(8.183+\rho)} \tag{7}$$

Equations (6) and (7) are applicable for members in tension (positive P) and compression (negative P). For practical applications, equations (6) and (7) give an excellent correlation to the expressions given by equations (2-5). However, for ρ other than the range of $-2.0 < \rho < 2.0$, the conventional stability functions should be used. The stability function approach uses only one element per member and maintains accuracy in the element stiffness terms and in the recovery of element end forces for all ranges of axial loads.

2.2. Stiffness Degradation Associated with Residual Stress

The Column Research Council (CRC) tangent modulus is employed here to account for gradual yielding effects due to residual stresses along the length of members under axial loads between two plastic hinges. The elastic modulus E (instead of moment of inertia I) is reduced to account the reduction of the elastic portion of the cross-section since the reduction of the elastic modulus is easier to implement than a new moment of inertia for every different section. Also, when this model incorporates appropriate geometrical imperfections, it may provide a very good comparison with the plastic zone solutions [8]. The CRC E_t may be written as [10]:

$$E_t = 1.0E \qquad \text{for } P \le 0.5P \tag{8}$$

$$E_t = 4\frac{P}{P_y}E\left(1 - \frac{P}{P_y}\right) \text{ for } P > 0.5P \tag{9}$$

where P_y; squash load and E; elastic modulus.

2.3. Stiffness Degradation Associated with Flexure

The CRC tangent modulus model is suitable for the member subjected to axial force, but not adequate for cases of both axial force and bending moment. A gradual stiffness degradation of a plastic hinge is required to represent the partial plastification effects associated with bending actions. The plastic hinge model to represent the gradual transition from elastic stiffness to zero stiffness associated with a fully developed plastic hinge the incremental force–displacement relationship may be expressed as equation (10) [12]:

$$\begin{Bmatrix} \overline{M_A} \\ \overline{M_B} \\ \overline{P} \end{Bmatrix} = \frac{E_t I}{L} \begin{bmatrix} \eta_A\left(S_1 - \frac{S_2^2}{S_1}(1-\eta_B)\right) & \eta_A\eta_B S_2 & 0 \\ \eta_A\eta_B S_2 & \eta_B\left(S_1 - \frac{S_2^2}{S_1}(1-\eta_A)\right) & 0 \\ 0 & 0 & A/I \end{bmatrix} \begin{Bmatrix} \overline{\theta_A} \\ \overline{\theta_B} \\ \overline{e} \end{Bmatrix} \tag{10}$$

Where S_1 and S_2 are stability functions, $\overline{M_A}$ and $\overline{M_B}$ are incremental end moments, $\overline{P}$ is incremental axial force, η_A and η_B are element stiffness parameters, $\overline{\theta_A}$ and $\overline{\theta_B}$ are incremental joint rotations, $\bar{e}$ is incremental axial displacement, A is area, I is moment of inertia, L is length of beam-column element and E_t is the tangent modulus.

The parameter η represents a gradual stiffness reduction associated with flexure at sections. The partial plastification at cross-sections at the end of elements is denoted by $0<\eta<1$. The η may be assumed to vary according to the simple parabolic expression as [11]:

$$\eta_i = \begin{cases} 4\alpha_i(1-\alpha_i) & \text{for} \quad \alpha_i > 0.5 \\ 1 & \text{for} \quad \alpha_i \le 0.5 \end{cases} \tag{11}$$

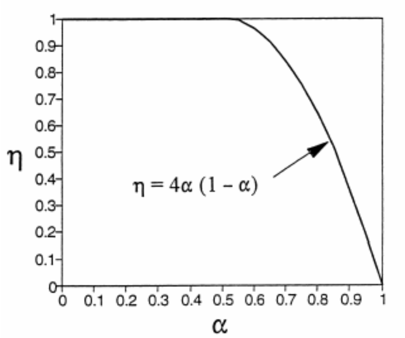

Figure 2. Gradual stiffness reduction associated with flexure

where α is the force–state parameter obtained from the limit state surface corresponding to the element end as [12]:

$$\alpha_i = \frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p} \quad \text{for} \quad \frac{P}{P_y} \ge \frac{2}{9}\frac{M}{M_p} \tag{12}$$

$$\alpha_i = \frac{P}{2P_y} + \frac{M}{M_p} \quad \text{for} \quad \frac{P}{P_y} < \frac{2}{9}\frac{M}{M_p} \tag{13}$$

Where P and M are second order axial force and bending moment at a section, M_p is the plastic moment capacity. It should be noted that:

1. When $0<\eta_A<1$ and $0<\eta_B<1$, the equation accounts for the effects of partial plastification at both ends of the element.
2. When $\eta_A=\eta_B=1$, both ends are fully elastic and the equation becomes conventional stiffness matrix including second order effects.
3. When $\eta_A=1$ and $0<\eta_B<1$, the equation represents the state at which end A is elastic, but end B is partially yielded.
4. When $0<\eta_A<1$ and $\eta_B=1$, end B is elastic, and end A is partially yielded.

It is important to demonstrate that only simple relationships for η are required to adequately describe the degradation in stiffness associated with distributed plasticity effects. Although more complicated expressions for η could be developed and proposed, a simple expression for η is needed for keeping the analysis model simple and straightforward. Also, the element model based on this approach should satisfy many aspects of the desirable attributes for inelastic beam-column elements outlined previously.

In Figure 3, the term α of 1.0 represents the plastic strength surface and α of 0.5 is assumed to be the initial yield surface which is assumed to have the same shape as the LRFD plastic strength surface [10]. As the α value varies from 0 to 0.5, the element end remains in the elastic state. When the α moves from 0.5 to 1.0, the element stiffness changes with a parabolic degradation shape shown in Figure 2.

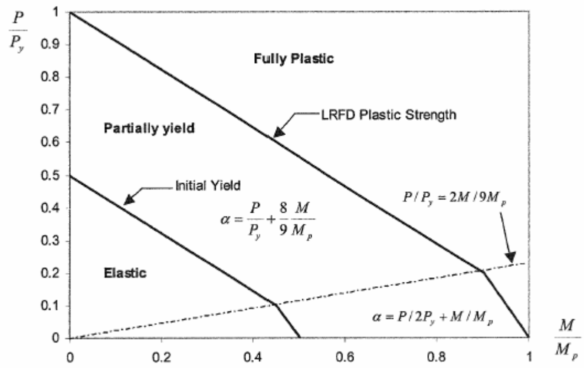

Figure 3. Plastic strength surface accounting partial yielding

3. INITIAL GEOMETRIC IMPERFECTIONS

There are two types of initial geometric imperfections for steel members: out-of-straightness and out-of-plumbness. Since these imperfections are also geometric nonlinear effects, they cause additional moments in the column members and then the member bending stiffness is further reduced. Combined with the use of CRC-E_t, one of three geometric imperfection models is used in the practical advanced analysis method: the explicit imperfection modeling method, the equivalent notional load method, or the further reduced tangent modulus method.

3.1. Explicit Imperfection Modeling Method

According to AISC Code of Standard Practice [13], the fabrication and erection tolerance for out-of-straightness is 1/1000 times the column length between braced points, and the maximum out-of-plumbness is limited to 1/500 times column length.

Since the out-of-straightness of a member should vary as a smooth curve, the imperfection shape is not known and many elements are needed in the analysis, the initial out-of-straightness of the column is ignored in this study. In other words, for simplifying the analysis, only modeling out-of-plumbness is considered for erection tolerance. The explicit imperfection modeling of an unbraced frame member is shown in Figure 4.

3.2. Equivalent Notional Load Method

The geometric imperfections of a frame may be replaced by equivalent notional lateral loads that are expressed as a fraction of the gravity loads acting on a story. Figure 5a shows an unbraced frame member with an out-of-plumbness of $L_c/500$. The out-of-plumb moment M_{IM} caused by the axial force P is $PL_c/500$ at the base. To determine the equivalent notional load accounting for the effects of initial geometric imperfection, consider a cantilever column shown in Figure 5b. The column is subjected to an axial force P and a lateral notional load nP at the top of the member. If the same out-of-plumbness of $L_c/500$ is assumed, Figures 5a and 5b must be equivalent and the base moment M_{NL} will be equal to M_{IM}. Thus, $(NP)(L_c)=PL_c/500$ or n=0.002 should be used. In this study, the proposed equivalent notional load for practical use is 0.002 times the total gravity loads applied on the considered story level. The notional load should be applied laterally at the top of each story.

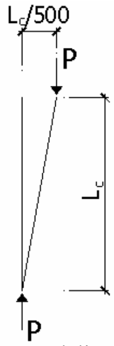

Figure 4. Explicit modeling method for modeling geometric imperfections of a member

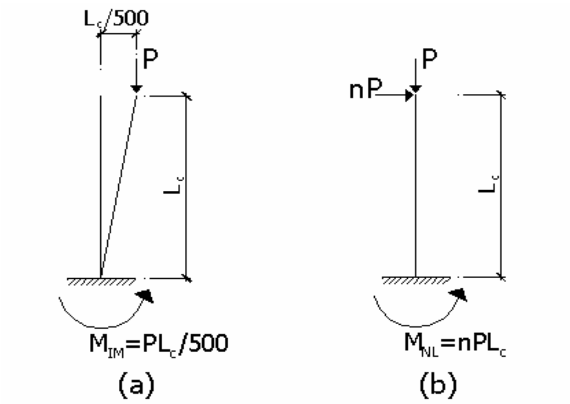

Figure 5. Equivalent notional load method for modeling geometric imperfections of a member

3.3. Further Reduced Tangent Modulus Method

Although explicit imperfection modeling method and the equivalent notional load method can well account for the effects of initial geometric imperfection, tedious work still exists. For both methods, the directions of the imperfections or notional loads should coincide with the directions of the deflections caused by the bending moments. If the directions of setting the imperfections or notional loads are not adequate, they may not weaken the structural system, instead, the structure will be strengthened. Usually, a trial and error process is used to determine the directions of the imperfections or notional loads for complicated systems.

To eliminate this tedious work, the further reduced tangent modulus approach was proposed [8]. By including the effects of stiffness degradation due to geometric imperfection, the reduction factor of 0.85, which is determined by calibrating the plastic-zone solutions [2], is used to further reduced CRC-E_t as given in equations (14-15). The same reduction factor of 0.85 is used for both braced and unbraced structures. The further reduced tangent modulus curve is shown in Figure 6.

$$E_t' = \xi_i E \quad \text{for } P \le 0.5P \tag{14}$$

$$E_t' = 4\frac{P}{P_y} E\xi_i \left(1 - \frac{P}{P_y}\right) \quad \text{for } P > 0.5P \tag{15}$$

Where $E_t^{'}$ is reduced tangent modulus, ξ_i is reduction factor for geometric imperfection.

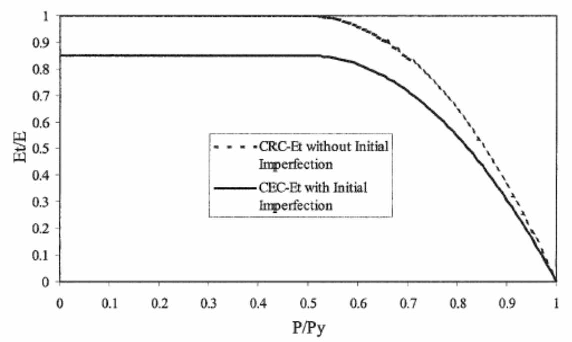

Figure 6. Further reduced tangent modulus method for modeling geometric imperfections

Although the reduction factor should be a function of the axial force in a column, a constant is used instead, because it allows for simplicity in the practical design process. Since the further reduced tangent modulus method can avoid the determination of the directions of the geometric imperfections and greatly reduce the input work that is necessary for the explicit imperfection modeling method and equivalent notional load method, it is recommended for practical steel frame design.

4. NUMERICAL EXAMPLE

A six story two bay frame has been proposed by Vogel [1] as a European calibration frame for nonlinear inelastic analysis. In this study, the refined plastic hinge analysis of the Vogel's calibration frame is investigated by using three different types of geometric imperfection models. Also, the refined plastic hinge analysis results are compared with the previous plastic zone analysis results.

4.1. Analyses of Vogel's Six Storey Calibration Frame

Both gravity and lateral loads are applied proportionally until failure occurs. All beams are continuously braced about their weak axes and all connections are assumed to be rigid. Columns are bent about the strong axes. The frame imperfection is assumed as 1/450 of story height. The modulus of elasticity is 205 kN/mm^2 and the yield stress is 235 N/mm^2. All frame members and loadings are given in Figure 7.

The plastic zone analyses results of the previous studies are given in Table 1 and the refined plastic hinge analysis of explicit imperfection modeling, notional load modeling and further reduced tangent modulus modeling methods are given in Table 2.

Table 1. Collapse load parameter of plastic zone analysis of Vogel's calibration frame

Reference	Analysis Type	Collapse Load Parameter, λ_u
Vogel [15]	Plastic Zone (Fiber Element)	1.11
Ziemian [16]	Plastic Zone (Fiber Element)	1.18
Clarke et al. [9]	Plastic Zone (Fiber Element)	1.17
Avery & Mahendran [3]	ABAQUS (Shell Element)	1.23

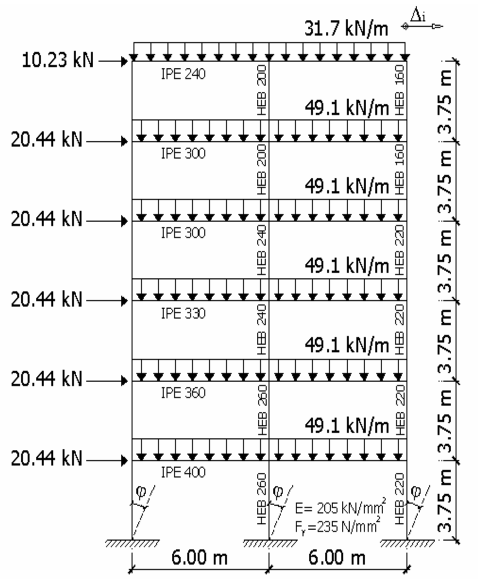

Figure 7. Vogel's six storey two bay calibration frame

4.2. Refined Plastic Hinge Analysis Results

Applied load ratio versus lateral displacement at roof level curves of explicit imperfection modeling, notional load modeling and further reduced tangent modulus modeling methods are given in Figure 8.

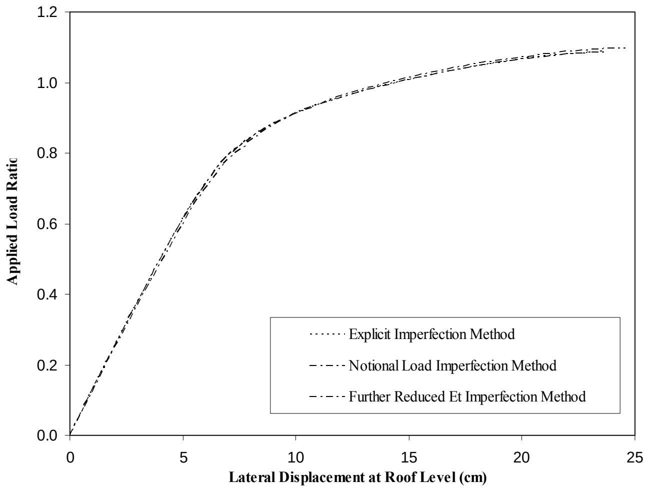

Figure 8. Comparison of load ratio - lateral displacement curves for different imperfection modeling by refined plastic hinge analysis of Vogel's two bay six storey calibration frame

Table 2. Refined plastic hinge analysis results for different imperfection modeling types

Analysis Type	Imperfection Modeling Type	Collapse Load Parameter, λ_u
Refined Plastic Hinge	Explicit Imperfection Modeling	1.09
Refined Plastic Hinge	Notional Loads Modeling	1.09
Refined Plastic Hinge	Further Reduced Tangent Modulus Modeling	1.10

5. CONCLUSIONS

It can be seen that the present results are in close agreement with the prediction by the plastic zone theory, implying the structure behavior can be accurately predicted by the simpler and more efficient method based on refined plasticity concept.

Each of three imperfection model give errors no more than 2% with respect to Vogel's plastic zone analysis results. Also, explicit imperfection modeling and notional load imperfection modeling detects equal ultimate load factors while further reduced tangent modulus imperfection modeling detects better result when compared with the plastic zone analysis results.

ACKNOWLEDGMENTS

Author wishes to appreciate the financial support of TUBITAK-BAYG.

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