σ Journal of Engineering and Natural Sciences
Mühendislik ve Fen Bilimleri Dergisi

**Sigma**
**2005/4**

# FUZZY OPTIMUM DESIGN OF PLANE TRUSS STRUCTURES

**Ömer CİVALEK***

*Akdeniz University, Civil Engineering Dept., Topçular-ANTALYA*

**Geliş/Received: 06.08.2004 Kabul/Accepted: 21.11.2005**

**ABSTRACT**

The main objective of structural engineers through out design history has been to obtain the optimum structure under the prescribed design conditions which can not only withstand external loads safely but also achieve an economic solution. The paper focuses on the use of the fuzzy set theory to optimum design of plane truss structures. The approach is illustrated on planar truss optimization problems and the results are discussed.
**Keywords:** Optimimum design, Fuzzy set theory, Plane truss structures.

**DÜZLEM KAFES YAPILARIN FUZZY OPTİMUM DİZAYNI**

**ÖZET**

Dizayn süresince yapı mühendisinin esas amacı, önceden tanımlanmış dizayn koşulları altında hem dış yüklere karşı dayanıma sahip ve hemde ekonomik bir optimum yapı elde etmektir. Bu çalışma, düzlem kafes sistemlerin optimum dizaynlarının fuzzy küme teorisi ile elde edilmesi ile ilgilidir. Yaklaşım, düzlem kafes yapıların optimizasyonu ile ilgili örnekler ile açıklanmış ve sonuçlar tartışılmıştır.
**Anahtar Sözcükler:** Optimum dizayn, Fuzzy küme teorisi, Düzlem kafes yapılar.

## 1. INTRODUCTION

As the optimization problems are great of importance in the field of structural engineering, numerous research works have been carried out and various algorithms can be mainly classified as the optimality criteria approaches and the mathematical programming techniques are detailed given in [1-5].

During the past ten years there has been a growing interest in algorithms, which rely on analogies to natural processes. The emergence of massively parallel computers made these algorithms of practical interest[6-12]. These well-known algorithms and techniques in this class include artificial neural networks, genetic algorithms, fuzzy logic, evolution algorithms and simulated annealing [22-29]. Although all these techniques have been adapted to the structural analysis, design and optimization problems, artificial neural networks (ANN) and fuzzy logic applications are widely used nowadays [13-20 ].

The main objective of structural engineering is to design structures which withstand external loads safely and at a minimum cost or weight [30-32]. During last decades, the developments in optimization methods which attempt to find the most economical solutions to

* e-posta: civalek@yahoo.com; Tel: (0542) 484 01 07

design problems by satisfying the required safety and rigidity constraints and minimizing the cost function, as well as the developments in nonlinear analysis methods which aim to determine the real behavior of structures under external effects give the structural engineer the opportunity to achieve this objective. Structural optimization is concerned with the computerized automatic design of structures which are optimum with respect to some major design parameter. In the structural engineering, this parameter has usually been structural weight, though cost or other factors are now being considered. The parameter being optimized is referred to as the objective function and the variables which can be changed to achieve the desired optimum are referred to as design variables. Mathematically this can be defined by saying that the problem is [32]

Minimize or Maximize $f(X) \quad X \in R^n$

Subject to the constraints $c_j(X) \le 0 \quad j = 1,...,m$

$$x_i^l \le x_i \le x_i^u \quad i = 1,...,n$$

where the design variables $X \in R^n$ are positive and the range of $X$ for which the constraints are not violated constitute the feasible region, $f(X)$ is the objective function to be minimized (maximized), $c_j(X)$ are the behavioral constraints, $x_i^l$ and $x_i^u$ are lower and upper bounds on a typical design variable $x_i$. Equality constraints are usually rarely imposed. Whenever they are used they are treated for simplicity as a set of two inequality constraints. If the objective function $f(X)$ is structural weight the design variables are size parameters such as bar cross-sections, plate thickness and, in certain cases, shape parameters which vary the geometrical configuration of the structures.

## 2. FUZZY MULTI-OBJECTIVE OPTIMIZATION

The fuzzy set theory defined by Zadeh [9] has been used to represent uncertian or noisy information in mathematical form. Fuzzy logic is an approximate reasoning method for coping with life's uncertainities. Occasionally, the characteristics of various systems are very difficult to describe with mathematical equations because of their complexity. In these cases, human experts may achive control by control values which are squeezed out from their long experience and represented by intuitive natural language.

It is not uncommon in civil engineering to divide the information avaliable for decision making into objective and subjective parts. The objective is discussed in countable information about the external world, while the subjective is concerned with the wisdom, experience and intuition of the engineer. In solving engineering mechanics problems, the loads, the material behavior and the system properties may be linguistically specified. For example, the load acting on the structure may be described with linguistic variables such as, severe, heavy or light. Further, the member's strength may be described using such as qualitative terms as highly stiff, flexible or very flexible. The fuzzy objective function and constrains are defined by their membership functions.

The fuzzy optimum design and fuzzy dynamic analysis of structures was considered by several researchers in the past [6-8,10,17]. The application of fuzzy sets theory to several civil engineering problems was reviewed by Brown and Yao [4]. More detailed information can be found in [4,5]. The conventional structural optimization problem can be given as Find $X$ which minimizes $f(X)$ Subject to

$$c_j(X) \le b_j \; ; j = 1,2,\ldots,m \tag{1a}$$

$$X \geq 0 \tag{1b}$$

where $b_j$ denotes the upper bound value on the constraint function $c_j(X)$ with $b_j \geq 0$. In the fuzzy approach, this problem can be defined as[4,8]

Find $X$ which minimizes $\tilde{f}(X)$

Subject to

$$c_j(X) \tilde{\in} \tilde{C}_j ; j = 1,2,\ldots,m \tag{2a}$$

$$X \geq 0 \tag{2b}$$

where ordinary upset $C_j$ denotes the allowable interval for the constraint function $c_j$, $C_j = [-\infty, b_j]$, and the wave symbols indicate that the operations or variables contain fuzzy information. If $d_j$ represents the permissible variation of $c_j(X)$ about $b_j$, then $\tilde{C}_j = [-\infty, b_j + d_j]$. The constraint $c_j(X) \tilde{\in} \tilde{C}_j$ means that $c_j$ is a member of a fuzzy subset $\tilde{C}_j$ in the sense of $\mu_{c_j}(C_j) \rangle 0$. The fuzzy feasible region is defined by considering all the constraints as

$$\tilde{S} = \bigcap_{j=1}^{m} C_j \tag{3}$$

Thus, the membership degree of any design parameters or vector *X* to fuzzy feasible region $\tilde{S}$ is given by

$$\mu_S(X) = \min_{j = 1,2,\ldots,m} [\mu_{C_j}(X)] \tag{4}$$

Namely, the minimum degree of satisfaction of the design vector *X* to all of constraints. A design vector *X* can be considered feasible provided $\mu_S(X) \rangle 0$ and the differences in the membership degrees of two design parameters $X_1$ and $X_2$ imply nothing but variation in the minimum degree of satisfaction of $X_1$ and $X_2$ to the constraints. Therefore, the optimum design or optimum solution will be a fuzzy domain *D* in $\tilde{S}$ with $\tilde{f}(X)$. The fuzzy domain *D* is described by

$$D = \{\mu_f(X)\} \cap \{ \bigcap_{j = 1,2,\ldots,m} \mu_{C_j}[c_j(X)]\} \tag{5a}$$

that is,

$$\mu_D(X) = \min\{\mu_f(X), \min_{j = 1,2,\ldots,m} [\mu_{C_j}(X)]\} \tag{5b}$$

where $\mu_D(X)$ and $\mu_{C_j}(X)$ denote the membership functions of the *i*th objective and *j*th constraint functions, respectively. If the membership function *D* is unimodal and has a unique maximum, then the optimum solution $X^*$ is one for which the membership function maximum

$$\mu_D(X^*) = \max[\mu_D(X)]; \quad X \in D \tag{6}$$

Let $f_{opt}$ be the optimum value of $f$ for the problem stated in Eq. (1) and $f_{opt} - \Delta f$ the optimum value of $f$ for the problem obtained by replacing $b_j$ by $b_j + d_j$ with $d_j \rangle 0$ , $j = 1,2,\ldots,m$ in Eq. (1). It is also noted that $f_{opt}$ is found with a tighter set of constraints, while $f_{opt} - \Delta f$ is found with a relaxed set of constraints. This is always possible since there will be lower $b_j$ and upper $b_j + d_j$ limiting values for each response quantity or constraints function $c_j(X)$ in the presence of fuzzy parameters. For computational convenience, the membership function of the objective is assumed to vary linearly between $f_{opt}$ and $f_{opt} - \Delta f$ , as indicated in Fig.1a [8]. Thus

$$\mu_f(X) = 1; \quad \text{if} \quad f(X) < f_{opt} - \Delta f \tag{7}$$

$$\mu_f(X) = 1 + \left(\frac{f_{opt} - \Delta f - f(X)}{\Delta f}\right); \quad \text{if} \quad f_{opt} \le f(X) \le f_{opt} - \Delta f \tag{8}$$

$$\mu_f(X) = 0; \text{ if } \quad f(X) > f_{opt} \tag{9}$$

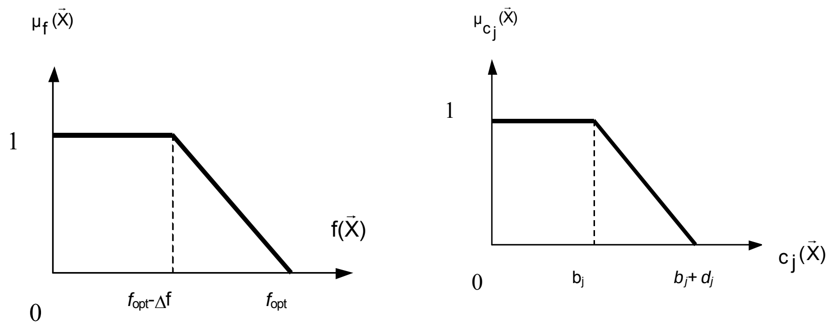


**Figure 1.** Membership functions for (a) objective and (b) constraints

As similar the objective function, the membership functions for the constraint (Fig 1b) can be defined as

$$\mu_{c_j}(X) = 1; \quad \text{if} \quad c_j(X) < b_j \tag{10}$$

$$\mu_{c_j}(X) = 1 - \left(\frac{c_j(X) - b_j}{d_j}\right); \quad \text{if} \quad b_j \le c_j(X) \le b_j + d_j \tag{11}$$

$$\mu_{c_j}(X) = 0 \text{; if } c_j(X) > b_j + d_j \text{; } j = 1,2,\ldots,m. \tag{12}$$

A simple flowchart for fuzzy optimization procedures is shown in Figure 2.

```
Start
  RETURN
      Give optimization parameters
      Give constraints
      Obtain fuzzy parameters
      Find X which minimizes f~(X)
  UNTIL (minimum error criteria is satisfied)
End.
```

**Figure 2.** A simple flow diagram for fuzzy optimum design

## 3. NUMERICAL EXAMPLES

*Example 1:* Consider minimum weight design of three-bar truss problem as shown in Fig. 3. This truss has frequently been used as an example in structural optimization literature. A detailed description of the formulation of the problem is given elsewhere [8] and only a final version is given here. Two different configurations as Case-I and Case-II are considered (Fig 3). The classical optimization problem of this problem is given as

Find $X = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$ which minimizes

$f_1(X) = 2\sqrt{2}\, x_1 + x_2$ and $f_2(X) = \frac{PL}{E}[1/(\sqrt{2}\, x_2 + x_1)]$

Subject to given displacements ($u \le u_i$) and stress($\sigma_1 \le \sigma^{max}$; $\sigma_2 \le \sigma^{max}$; $|\sigma_1| \le \sigma^{min}$) constraints. For Case-I: P =20N, $\rho$ = 1 N/cm$^3$, $\sigma^{max}$ = 20 N/cm$^2$, $\sigma^{min}$ = -15 N/cm$^2$, $X_i$(max) =5 cm$^2$, $X_i$(min) =0.1 cm$^2$, $L$ = 100 cm. Where $E$ is Young's modulus, $L$ the semiwidth of the truss, and $P$ is the load. Fuzzy optimization problem are given

Find $X = \begin{Bmatrix} \lambda \\ x_1 \\ x_2 \end{Bmatrix}$ which maximizes $f(X) = \lambda$

Subject to (for stress)

$$\lambda \ge \left\{ \frac{3(2\sqrt{2}\, x_1 + x_2) - 5\sqrt{2}}{\sqrt{2}} \right\} - 1$$

$$\lambda \ge \left\{ \frac{(x_2 + \sqrt{2}\, x_1) - (\sqrt{2}\, x_1^2 + 2\, x_1 x_2)}{0.2(\sqrt{2}\, x_1^2 + 2\, x_1 x_2)} \right\} - 1$$

$$\lambda \geq \left\{ \frac{1-(x_1+\sqrt{2}\,x_2)}{0.2(x_1+\sqrt{2}x_2)} \right\} - 1$$

$$\lambda \geq \left\{ \frac{4x_2 - 3(\sqrt{2}\,x_1^2 + 2\,x_1 x_2)}{0.6(\sqrt{2}\,x_1^2 + 2\,x_1 x_2)} \right\} - 1$$

$$\lambda \geq \left\{ \frac{2-(x_1+\sqrt{2}\,x_2)}{0.2(x_1+\sqrt{2}x_2)} \right\} - 1$$

$$\lambda \geq \left\{ \frac{2-(x_1+\sqrt{2}\,x_2)}{0.2(x_1+\sqrt{2}x_2)} \right\} - 1$$

(for displacement of the loaded join)

$$\lambda \geq [\frac{0.1 - x_i}{0.02}] - 1; \quad i = 1,2$$

The results of this problem is given by $\lambda = 0.53$ ; $x_1 = 0.61$ cm$^2$ and $x_2 = 0.86$ cm$^2$ with $f = 2.5854$ . Where f is the aim function for fuzzy optimization problem. For Case-II given data: P =50000 N, $u_1 \leq 0.05$ cm, $u_2 \leq 0.1$cm, $L_1$= 175cm, $L_2$ = 200cm, $L_3$ = 250 cm. Fuzzy optimum design of truss is obtained as: $x_1 = 74.92$ cm$^2$, $x_2 = 22.22$ cm$^2$ and $x_3 = 40.15$ cm$^2$ with $f = 306.7$ .

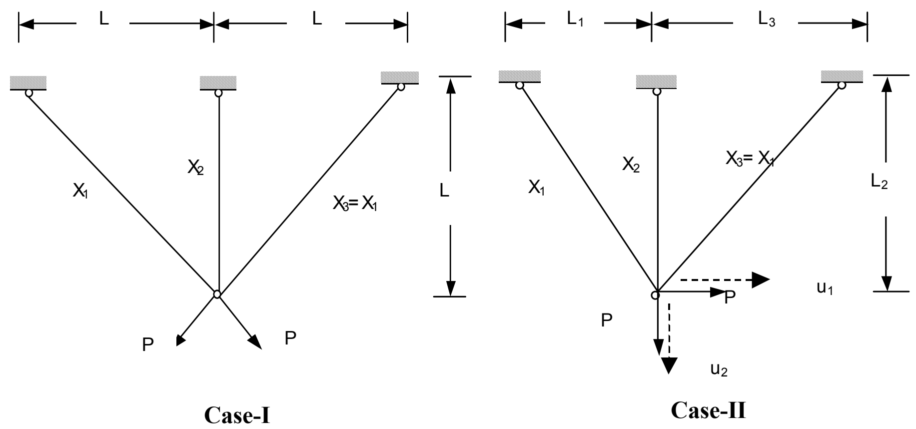


**Figure 3.** Three-bar planar truss

*Example-2:* The Ten-bar truss problem is considered (Figure 4). This truss has been extensively used as a test problem in structural optimization literature. The design variables of for this problem were the areas of members. Material properties, stress and displacement constraints and minimum areas used for this example : Modulus of elasticity, E = 6.895×10$^4$ MPa, L= 914.4 cm, P = 445.37 kN, ρ= 0.027 N/cm$^3$, $\sigma_i \leq 172.25$ Mpa (j =1,2,…,10), $u_i \leq 5.08$ cm (i =1,2,3,4).

The fuzzy optimization results are given in Table 1. Classical optimization [21,30] results are also given in this table for comparison.

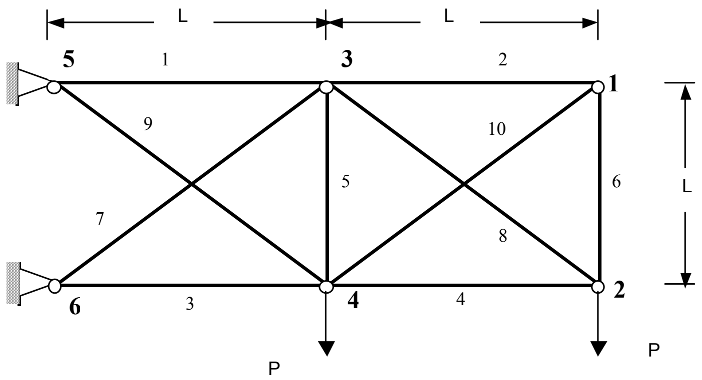


**Figure 4**. Ten- bar planar truss structures

**Table 1.** Comparison of the results

| | Design variables [cm$^2$] | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | $X_9$ | $X_{10}$ |
| Ref.30 | 51.14 | 0.645 | 51.98 | 25.41 | 0.645 | 0.645 | 35.93 | 35.93 | 37.08 | 0.645 |
| Ref.21 | 51.14 | 0.65 | 51.95 | 25.38 | 0.65 | 0.65 | 36.02 | 36.02 | 37.12 | 0.65 |
| This Study | 52.21 | 0.661 | 53.01 | 24.26 | 0.661 | 0.661 | 35.28 | 35.28 | 38.50 | 0.661 |

*Example 3:* Consider minimum weight design of a statically determinate nine-bar truss problem under a specified loading conditions as shown in Figure 5. Constraints were imposed by considering member vertical displacement ($u_1$ and $u_3$) and horizontal displacement ($u_2$).

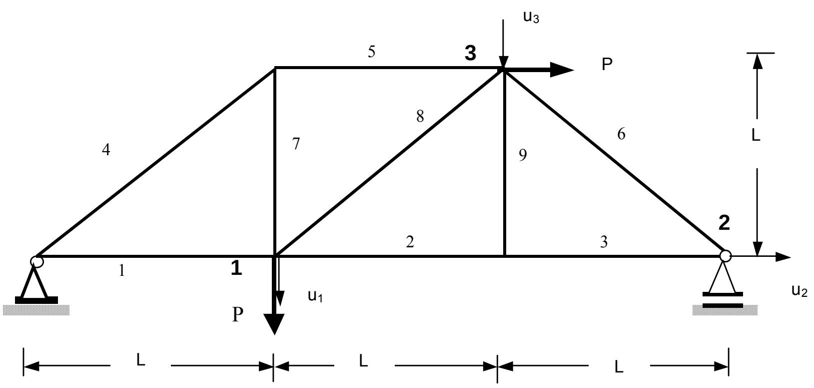


**Figure 5.** Nine- bar planar truss structures

A detailed description of the formulation of the problem is given elsewhere [21] and only a final version is given here. Displacement constraints are 0.1 cm at nodes 2 and 3 in horizontal and vertical directions, and 0.2 cm at nodes 1 in vertical directions. Given data is applied load, P =30000 N, and member length, L= 100 cm. The obtained results are summarized in Table 2 with the results given by Kanarachhos et al. [21] by the optimality criteria. It is shown that, the results compare very well with the solution of Kanarachhos et al. [21].

**Table 2.** Comparison of the results for Nine-bar truss

| | Cross sectional area [cm$^2$] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ |
| Optimality Criteria [Ref.31] | 46.10 | 33.09 | 33.11 | 23.11 | 17.1 | 33.11 | 18.10 | 20.21 |
| Fuzzy Optimization [This Study] | 44.25 | 35.96 | 36.01 | 21.52 | 16.85 | 36.01 | 19.47 | 21.35 |

## 4. CONCLUDING REMARKS

The optimizaton of plane truss structures containing fuzzy information has been considered. The results found by using fuzzy theory are sufficiently close to the results obtained by classical optimization theory. The illustrative numerical examples show that the fuzzy set theory seems to be a rational and effective approach for optimization of structures. Optimization using fuzzy sets theory was seen to be faster than the classical theory. On the other hand, the required computing capacity has relatively decreased. Mathematically, considering multiple constraints do not cause difficulties as with the classical optimization approach. The present approach can be used whenever the reliability of the structure is specified with respect to several criteria such as stress, deflection, buckling and natural frequency of vibration.

## REFERENCES


[1] Venkayya, V.B., “Design Of Optimum Structures, Computers and Structures”,1 pp 265-309,1971.

[2] Kirsch, U., Taye, S., “Structural Optimization In Design Planes”, Computers and Structures, 31(6), pp 913-920,1989.

[3] Kirsch, U., Benardout, D., “Optimal Design Of Elastic Trusses By Approximate Equilibrium”, Comput. Meth. App. Mech. Eng., 22, pp 347-359, 1980.

[4] Ross, T. J., “Fuzzy Logic With Engineering Applications”, McGraw-Hill, Inc., 1995.

[5] Brown, C.B., and Yao, T.P., “Fuzzy Sets And Structural Engineering”, Journal of Structural Eng., ASCE, 109(5), 1211-1225,1983.

[6] Yuan, W.G., Quan, W.W., “Fuzzy Optimum Design Of Structures”, Engineering Optimization, 8, 291-300,1985.

[7] Yuan, W.G., Quan, W.W., “Fuzzy Optimum Design Of Aseismic Structures”, Eartquake eng. Struct. Dyn., 13, 827-837, 1985.

[8] Rao, S.S., “Multi-Objective Optimization Of Fuzzy Structural Systems”, Int. J. for Num. Meth. Eng., 24, 1157-1171,1987.

[9] Zadeh, L., “Fuzzy Sets”, Information and Control, 8, 338-353, 1965.

[10] Civalek, Ö., “Earthquake Resistant Structural Design by Neoro- Fuzzy Technique”, Forth National Earthquake Engineering Conference, 17-19 September, Ankara, 1997.

[11] Civalek, Ö., “Flexural And Axial Vibration Analysis Of Beams With Different Support Conditions Using Artificial Neural Networks”, International Journal of Structural Engineering and Mechanics, 18 (3),303-314, 2004.

[12] Ülker, M., Civalek, Ö., "The Buckling Analysis Of Axially Loaded Columns With Artificial Neural Networks", Turkish J. Eng. Env. Sci., TUBITAK, 26, 117-125,2002.
[13] Ülker, M., Civalek, Ö., "The Analysis of Circular Cylindrical Shells by Hybrid Artifcial Intelligence Technique", Technical Journal of Turkish Chamber of Civil Eng., 12(2), 2401-2417,2001.
[14] Civalek, Ö., "The Analysis Of Time Dependent Deformation In R.C. Members By Artificial Neural Network", Journal of Eng. Sciences of Pamukkale Univ., 3(2),331-335 1997.
[15] Civalek, Ö., "The Analysis of Circular Plates by Neoro- Fuzzy Technique", Journal of Eng. Science of Dokuzeylül University, Vol. 1(2); 13-31,1999.
[16] Civalek, Ö., "The Analysis Of The Rectangular Plates Without Torsion Via Hybrid Artificial Intelligent Technique", Proceedings of the Second International Symposium on Mathematical & Computational Applications, September 1-3, Azerbaijan, 95-101;1999.
[17] Civalek, Ö, "The Analysis of Beams on Elastic foundation by the Method of Neoro-Fuzzy", 7th. National soil mechanics and foundation engineering conferences, 22-23 October, Yıldız Univ., Istanbul, 1998.
[18] Civalek, Ö., "Fuzzy or Hazzy Logic", TÜTEV Technical Journal, 2004 (in press).
[19] Civalek, Ö., "Artificial Intelligent-Conversation by Ömer CİVALEK", Turkish Chamber of Civil Eng.,Engineering News(TMH), Vol. 423, 40-50, 2003.
[20] Rajeev, S., Krishnamoorthy, C.S., "Genetic Algorithms-Based Methodology For Design Optimization Of Truss", Journal of Structural Eng., ASCE,123, pp. 350-358, 1997.
[21] Haug, E., and Arora, J., "Applied optimal design", John Wiley and Sons, 1979.
[22] Erbatur, F., Hasançebi, O., Tütüncü, İ., and, Kılıç, H., "Optimal Design Of Planar And Space Structures With Genetic Algorithms", Computers and Structures, 75, 209-224, 2000.
[23] Zurada, J. M., "Introduction To Artificial Neural Networks", West Publishing Com.,1992.
[24] Zadeh, L.A., "Fuzzy Sets As a Basis For A Theory Of Possibility", Fuzzy Sets and Systems,1(1), 3-28, 1978.
[25] Rojas, R., "Neural networks, A Systematic Introduction", Springer, Germany,1996.
[26] Goldberg, D.E., "Genetic Algorithms In Search Optimization And Machine Learning", Addison-Wesley, MA, 1989.
[27] Adeli, H. and Hung, SL., "Machine Learning- Neural Networks, Genetic Algorithms And Fuzzy Systems", John Wiley & Sons, Inc, 1995.
[28] Ghaboussi, J., Garrett, Jr., Wu, X. "Knowledge- Based Modeling Of Material Behavior With Neural Networks", Journal of Structural Engineering, ASCE, 117( 1), 132-153,1991.
[29] Hajela, P., Berke, L., "Neurobiological Computational Models In Structural Analysis And Design", Computers and Structures, 41(4), 657-667, 1991.
[30] Schmit, A., and Miurai H., "Approximation Concepts For Efficient Structural Synthesis", NASA CR-2552, Univ of California, Los Angeles, CA, 1976.
[31] Kanarachos, A., Makris, P., and Koch, M., "Localization of Multi-Constrained Optima and Avoidance of Local Optima in Structural Optimization Problems", Comput. Meth. In Appl. Mech. And Eng., 51, pp.79-106,1985.
[32] Saka, M., Ülker, M., "Optimum design of geometrically nonlinear space trusses", Computers and Structures, 41(6), 1387-1396,1991.