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STRESS DISTRIBUTION OF THE CHAIN LINK BY MEANS OF BOUNDARY ELEMENT AND FINITE ELEMENT METHODS

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ABSTRACT

Finite and boundary element methods are the well known and the frequently used methods of continuum mechanics. The theoretical differences and the superiorities of the techniques over each other are widely investigated in literature. In the study, roller chains which are used as pulling and driving members of materials handling mechanisms are inspected. Stress analysis of a standard roller chain link is performed using both boundary and finite element methods. The mechanical behaviors of a standard roller chain which is loaded by the maximum allowed load are considered. Comparing the results of the both techniques with each other and the results of literature, the appropriate method for the roller chain problem is proposed.
Keywords: Chain, boundary element method, finite element method, stress distribution.

SINIR ELEMAN VE SONLU ELEMAN METOD YARDIMIYLA ZİNCİR BAKLASININ GERİLME DAĞILIMI

ÖZET

Sonlu eleman ve sınır eleman metodları, sürekli ortam mekaniği problemlerinin incelenmesinde yaygın olarak kullanılan metodlardır. Her iki metodun birbirinden teorik farklılıkları ve üstüklükleri, literatürde yaygın olarak incelenmektedir. Bu çalışmada, transport makinalarının çekme ve tahrik elemanı olarak kullanılan makaralı zincirler ele alınmıştır. Standart makaralı zincir baklasının gerilme analizi, hem sınır eleman hem de sonlu eleman metodlarıyla gerçekleştirilmiştir. Maksimum yükleme halindeki standart makaralı zincirin mekanik davranışları gözönüne alınmıştır. Her iki metodun sonuçları birbirileriyle ve literatürdeki sonuçlarla karşılaştırılarak, makaralı zincir probleminin çözümü için en uygun metod önerilmiştir.
Anahtar Sözcükler : Zincir, sonlu eleman metodu, sınır eleman metodu, gerilme dağılımı.

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1. INTRODUCTION

Roller chains are widely used as pulling and driving members of chain mechanisms in escalators, passenger conveyors and especially in conveyors. The most common breakdown of the chain mechanisms occurs by joint erosion. The mechanism becomes useless if the length of the chain extends more than the allowable maximum value of the length because of the erosion. Time endurance of the link sheets with shaft and pulleys with rings are restricted by being broken or cracked of the members by fatigue phenomena affecting dynamical loads. Considering the chain

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erosion fundamental principal, the design of the chain mechanisms and the selection of chain type are standard procedures. Only the calculation of the erosion appears in the standards since the fatigue influence appears after the erosion influence at the mechanisms, appropriately designed [1,2].

Roller chains are manufactured by combining various shaped sheet tablets which are named as lamella made of St60 steel with pins (St50). It is possible to use poly row lamellas to increase the endurance for the breaking. Also, chains are manufactured like hollow curved lamellas by hollowing the middle edges, for the aim for lightening the construction; or flat lamellas [3].

The stress analysis of the chain links by the aids of the triangle finite elements has been examined and compared with the photo-elastic results by [4]. Several holes to decrease the stress collapses on the chain links for his experimental study has been made by [5]. Besides these studies, several experimental and analytical researches about the load distribution and the lubrication effect of the double pitched steel roller chains on the steel sprockets has been examined by [6]. In the study of Abdulaliyev and Toprak, theoretically, an optimum design criteria has been developed in order to have uniformly distributed stress which has been less than the allowable stress of the material chain plate, and they have calculated the normal force, the bending moment and the normal stress variations around the pin hole. The stresses have been analyzed due to distributed external load, instead of simple concentrating tensile force. Some changes on the shape (classical geometry) have been found for material saving and increasing the strength [7]. The elasto-plastic stress analysis of the composite roller chain link plates is carried out two dimensional finite element analysis by [8].

In the study, the Gall typed roller chains which are used as pulling and driving members of materials handling mechanisms are inspected and the mechanical behaviors of a standard roller chain which is loaded by the maximum allowed load are considered. Stress analysis of a standard roller chain link is performed using both boundary element and finite element methods. Thus the practically comparison of the popular methods became possible. Comparing the results of the both techniques with each other and the results of literature, the appropriate method for the roller chain problem is proposed. The chain link with the dimensions which is chosen for stress analysis with the loading conditions is illustrated in Figure 1.

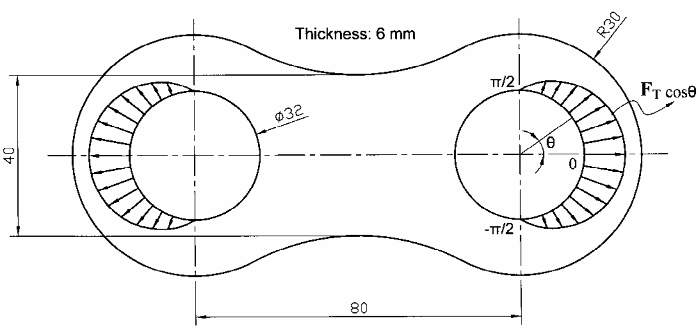


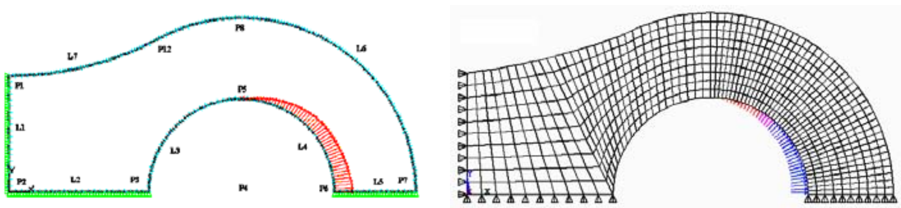
Figure 1. The chain link with fully dimensioned

2. TWO-DIMENSIONAL MODELLING TECHNIQUES FOR FEM AND BEM

The most popular and well known methods for engineering applications are finite element method (FEM) and boundary element method (BEM). Numerical methods are based on the principal that is possible to derive some equations and relationships that describe accurately the behavior of small differential part of the body. By dividing the entire body into a large number of these small 'parts' and using further relationships to link up or assemble these parts together, it is possible to obtain a reasonable accurate prediction of the values of variables such as stresses and displacements in the body [9].

The basic principal of all the numerical methods is dividing all the large structure into small elements having simple shapes. The unknown variables of an element are the displacement values for each nodal point. Boundary element and finite element software combine the shape functions defining these small elements to get the matrix system of the whole structure. The unknown displacement values are calculated considering the known loads and boundary conditions, and then stress values of each element are computed using the deformations of the nodal points [10].

The major difference between the boundary element method and the others is that, boundary element method only divides and analysis the boundaries of the interested domain. The technique can compute stress concentrations and deformations, which may form because of the geometry of the structure, sensitively in a short time interval. The basic differences between the discretization of the roller chains by boundary element and finite element methods are shown in Figure 2 [11].



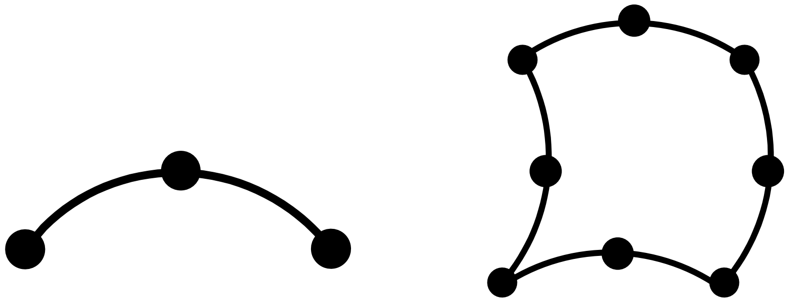
(a) BEM model (b) FEM model

Figure 2. Chain link model in numerical methods

Any stress or deformation distributions along the thickness direction of the chain link are not expected since no loads are acted along to the thickness direction. In other words it is enough to enter 2D model to the software and only inspect 2D analysis. As the chain link has symmetry along its vertical and horizontal axis, quarter model of the link as shown in Figure 2 is enough to enter to the computer programs [12].

The types of elements used for 2D and axisymmetric boundary element problems are line elements. But note that their form also may be like curve. Triangular or rectangular shaped elements may be used for same sort of finite element problems. The finite element link model is modeled with rectangular elements since discretizing a model like chain links which have lots of curved boundaries with triangular elements is not recommended. Each element represents the potential and stress behavior of a small part. There are different types of elements which the designer may select; such as constant, linear and quadratic. Also each element has several nodes which are used to calculate the variables of the problem. The number of nodes depends on the degree of the element. The values of the variables of the any position on the element are computed by interpolation among the nodes. Generally accuracy of the reason increases while the number of the nodal points increasing in the model; but this also expands the numbers of the calculation matrixes and calculation time for the computer [13,14]. The quadratic elements which are used in analysis are schematically illustrated in Figure 3. The modeled chain links which may be seen in Figure 2, are divided into 189 boundary elements and 700 finite elements.

The major superiority of boundary element method to finite element method comes form its sufficiency of discretization of only the areas of the interested volume instead of the whole volume for three-dimensional problems and only the boundaries of the interested area instead of the whole area for two-dimensional problems. Hence boundary element codes are easier to use with existing solid modelers and mesh generators. This advantage is particularly important for designing as the process usually involves a series of modifications which are more difficult to carry out using finite elements. Meshes can easily be generated and design changes do not require a complete meshing [15].



Boundary element (line element) Finite element (rectangular element)

Figure 3. Quadratic elements used in BEM and FEM

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3. STRESS ANALYSIS OF THE CHAIN LINK

Numerical techniques consist of three basic steps: "pre-processor, processor and post-processor". The pre-processing stage consists of the procedures as constituting the geometry of the numerical procedure model, entering the physical and material properties of the model to the software and revealing the load acting on model [12]. The boundary element stress analyses are performed using BEASY® and finite element analysis are performed using ANYSY® software.

The two-dimensional chain link geometries entered to boundary element and finite element analyze software are illustrated in Figure 2, respectively. The material properties, such as Young's module ($2.1 x 10^5$ N/mm^2) and Poisson ratio (0.3) of the chain link are entered the programs, and also supporting and load conditions as boundary conditions of the model are entered to the software. The supporting conditions of the model are designed such ways: Vertical edge of the model is clamped along x axis and horizontal edges are clamped along y axis. In the conventional design, external force is assumed to be as a simple tension force on the axisymemetrical direction. However, in practice this assumption does not reflect the force application on the chain link. Therefore, there is a distributed load along the half area of the pin hole as shown in Figure 1. The form of distributed force can be taken as $F = F_T \cos\theta$ between $-\pi/2 \le \theta \le \pi/2$ interval where the maximum load, acted on the chain link [13]. Kesikçi, 2003].

The boundary conditions which are entered the numerical method software of the chain link are also illustrated in Figure 2.

The contour plots of Von Mises and Maximum Principal stress distributions on the chain link and the maximum values of the stresses which are computed using quadratic elements may be seen in Figure 4, separately for boundary element and finite element analysis.

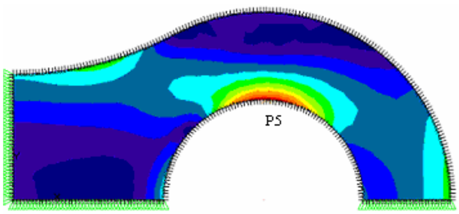
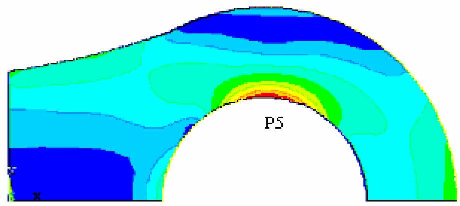
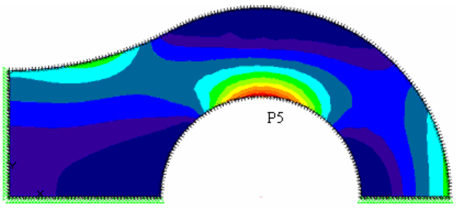
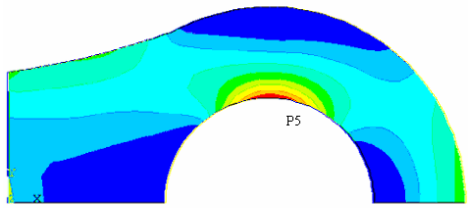
Boundary element analysis	Finite element analysis
 231.98 N/mm^2	 224.98 N/mm^2
(a) Von Mises stresses	
 231.61 N/mm^2	 224.78 N/mm^2
(b) Maximum Principal stresses	

Figure 4. Stress distributions on the chain link

The maximum Von Mises and Maximum Principal stress values which are calculated performing boundary element analysis are respectively 231.98 N/mm^2 and 231.61 N/mm^2 and the values calculated performing finite element analysis are respectively 224.98 N/mm^2 and 224.78 N/mm^2. It is realized that the maximum stress areas locate in the boundary of the pin hole, at the point P5 and the vicinity of the point. It may also recognized that the maximum stress and the force flow boundaries (stress contours) are recognized in the same way. The difference of the computed stress values by both methods are 3% and the calculated difference allows to comparing the results. The small differences of the analyze results are originated form the software of the analyze programs and differences of the regions of the stress values.

For the comparable level of accuracy, boundary element method requires less number of quadratic elements and nodal points. For boundary element analysis 189 quadratic elements and 384 nodal points are used, however 700 quadratic elements and 2261 nodal points are used for finite element analysis. Therefore for the same level of accuracy, finite element method requires 3.7 times more elements and 5.8 times more nodal points than boundary element method.

For boundary element method, discretization and constituting elements all over the interested domain is observed easier than the finite element method since boundary element method only divides the boundaries. Also it may be observed that the sizes of the boundary elements are smaller than finite elements.

Since the maximum stress region is computed as the region near the point P5 by both method, the vertical section which starts from the point P5 and continues along +y axis may be realized as the most critical section for investigation. The stress distribution of the section is shown in Figure 5 by graphically with the results of boundary element and finite element analysis separately.

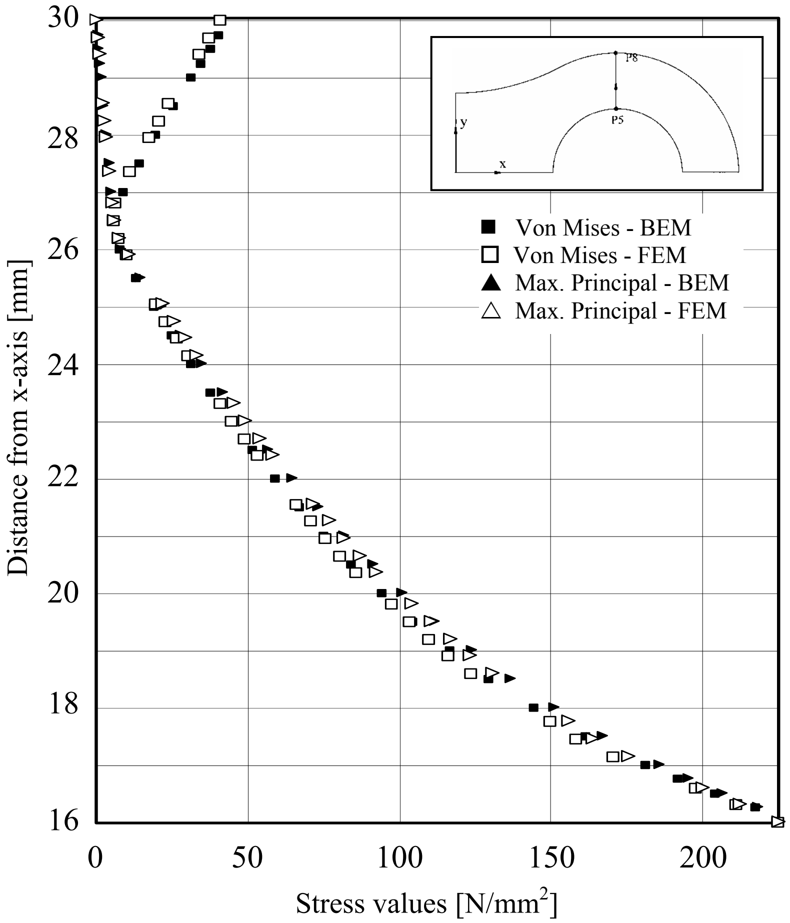


Figure 5. Stress distribution of the most critical section (P5-P8)

It may be recognized that the stress values of both boundary and finite element methods are very close to each other. The maximum value of Von Mises and Maximum Principal stresses are appeared at the point P5 which locates in the boundary of the pin hole and while the values of Maximum Principal stresses are diminished along the +y axes and arrived at zero value for the point which locates in the outer boundary of the chain link, the values of Von Mises stresses are made a minimum pig at the point with coordinates (40,26.5) and then increased continuing to move outer boundary of the chain link.

The central part of chain link is more rigid comparing to circular part. As a result of this the model of circular part may be considered as curved beam which is fixed from both ends [7]. Considering the chain link application as a problem of "bars with curved axis", the analytical solution of the problem is performed for the critical cross-section of the chain link, to manage the comparison of the numerical methods. The geometry of the critical cross-section is illustrated in Figure 6. Normal stress at the point of the critical cross-section can be written in terms of bending moment as:

$$\sigma = \frac{M}{F \cdot k} \frac{(r - a/2)}{a/2} \tag{1}$$

where M is bending moment ($M = Q \cdot r$), F is cross-sectional area of chain link ($F = h \cdot t$), r is the radius which defines the neutral plane, and k is the distance between geometric center G and neutral center O ($k = R - r$) as shown in Figure 6, $a/2$ is the radius of the pin hole.

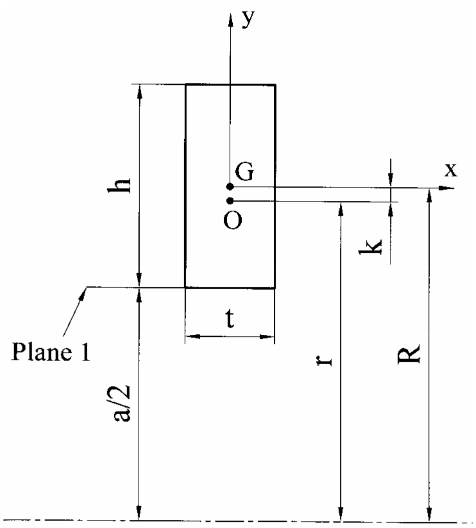


Figure 6. Geometry of the critical cross-section of chain link

The radius defining the neutral plane can be found in the following form

$$r = \frac{h}{\ln \frac{R + h/2}{R - h/2}} \tag{2}$$

where h is the height of critical cross-section, and R is the radius which defines the geometric plane.

The normal maximum stress has been calculated for the example on which numerical values are Q = 1667 N, h = 16 mm, t = 6 mm, and a = 32 mm. The maximum normal stress is calculated as 238.1 N/mm^2 for safety factor of 2.5 in respect of standards for analytical solution.

4. CONCLUSIONS

Pulling and driving members of escalators, roller chains are considered in this study. Two dimensional geometrical model of the chain link is formed and stress analyses are performed using both boundary element and finite element methods.

Since the maximum calculated stress is estimated less than the yielding point of the material St60 (335 N/mm^2), the chain link is remained in elastic region although the maximum load is applied in respect of standards. Considering the ultimate strength of the material as 600 N/mm^2, the safety factor of the chain link is calculated as 2,586 from boundary element analysis and 2.666 from finite elements. Although the values are very close to each other; the results of boundary element method may be accepted as more accurate results, since the safety factor is assumed as 2.5 for standard calculation of the chains. Boundary element solution (231.98 N/mm^2) is closer to the analytical solution (238.1 N/mm^2) than the Von Mises result of the finite element analysis (224.98 N/mm^2), it is clearly understood that BEM is more suitable than FEM for the chain link application. Although the maximum stress areas are appeared on the boundaries of the pin hole, the boundary element method is more appropriate for the chain link application than the finite element method.

5. ACKNOWLEDGEMENT

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