YIELDING ANALYSIS FOR SHORT COLUMNS WITH CONFINED CONCRETE HAVING CIRCULAR SECTION CONSIDERING STRAIN HARDENING IN REINFORCEMENT

Bilge DORAN, Sema NOYAN ALACALI , Ayhan ÜNAL

Yıldız Teknik Üniversitesi, İnşaat Fakültesi, İnşaat Mühendisliği Bölümü, Yıldız-İSTANBUL

Geliş Tarihi: 09.06.2003

KUŞATILMIŞ KESİTLİ DAİRESEL BETONARME KOLONLARIN DONATIDAKİ PEKLEŞME GÖZÖNÜNE ALINARAK AKMA LİMİTİNE GÖRE ÇÖZÜMLENMESİ

ÖZET

Betonarme yapıların limit tasarımında potansiyel plastik mafsal bölgelerinin plastik dönme kapasiteleri, kabul edilen hesap momentleri dağılımının gerçekleşebilmesini sağlayacak ölçüde olmalıdır. Plastik dönme kapasiteleri, bu plastik kesimlerdeki akma momentleri ve eğrilikleriyle ilişkilidir. Bu bağlamda, kuşatılmış kesitli betonarme dairesel kolonlarda donatıdaki pekleşme gözönüne alınarak akma yükü, moment ve eğriliklerinin belirlenmesiyle ilgili tasarım algoritmaları geliştirilebilir.

ABSTRACT

In limit design of reinforced concrete structures, plastic rotation capacities of the potential plastic hinge regions must be sufficient for the development of assumed distribution of the design bending moments. The plastic rotation capacities are related with the yield moments and curvatures in these regions. In this context, based upon an appropriate steel behaviour model including strain hardening, a reliable and accurate algorithm can be developed for confined circular column sections.

1. INTRODUCTION

Ductility of RC members subjected to bending and axial load is functions of the ultimate and yielding limit curvatures. The equations which define the rotation capacities of potential plastic hinges are related to the ultimate moment, yielding moment and curvatures [11]. The algorithms for the design of short columns with confined concrete having circular section considering strain hardening in reinforcement have been derived in Ref.2. In this study, the algorithms have been formed for the evaluation of the yielding limit loads, moments and curvatures for the confined concrete columns having circular sections.

The eccentricity of the axial load, $(e = M / N)$, defines the failure type for a column section. In general case, the type of failure depends on whether e is less than or greater than e_b (balanced eccentricity). If $e < e_b$ compression failure, $e > e_b$ tensile failure occurs.

2. CONSTUTIVE MODELS FOR THE MATERIAL

The section ductility of RC beams and columns is known as the ratio of ultimate curvature (ϕ_u) to yielding limit curvature (ϕ_y): $\mu = \phi_u / \phi_y$ (Fig.1). Yielding limit (L_y) and ultimate limit (L_u) can be obtained from stress-strain behaviour models of the concrete and steel materials (Figs.2 and 3) [2,12,14,15].

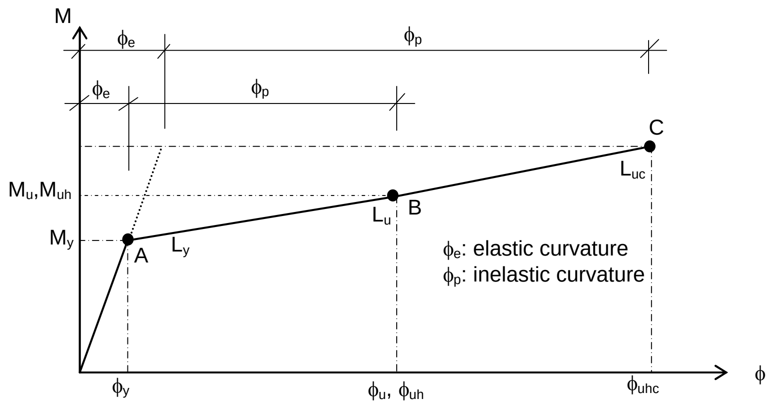


Figure 1. Moment-curvature relationships for the confined and unconfined sections

2.1. Behaviour Model for Concrete

Based on the existing experimental evidence, stress-strain behaviour models have been proposed for concrete unconfined and confined by circular spirals [5,7,8,12,14,16].

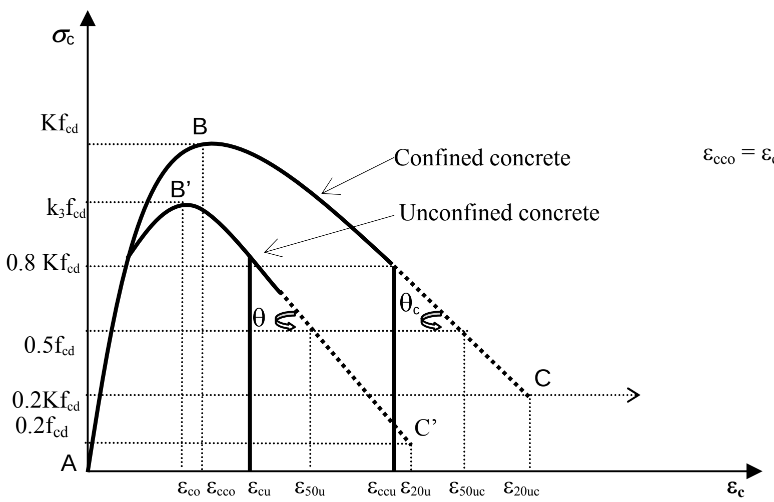


Figure 2. Stress-strain behaviour model for concrete confined by circular spirals

Characteristic cylinder compressive strength of the concrete confined with circular spirals can be determined as [10]

$$f_{cck} = f_{ck} + 2.05\rho_h f_{ywk} = Kf_{ck}\ ;\ K = f_{cck} / f_{ck} = f_{ccd} / f_{cd} \quad (1)$$

K can be expressed as follows,

$$K = 1 + 2.05\rho_h (f_{ywk} / f_{ck}) \text{ for C< C50} \quad (2a)$$

$$K = 1 + 1.5375\rho_h (f_{ywk} / f_{ck}) \text{ for C } \geq \text{ C50} \quad (2b)$$

where K is the confinement coefficient, f_{ck} is the characteristic cylinder compressive strength of the unconfined concrete, f_{ywk} is the characteristic yield strength of the spiral bar, f_{cd} and f_{ccd} are the design compressive strengths concrete unconfined and confined, respectively.

The ratio of volume of spiral bar to volume of concrete core measured to center lines of spirals is

$$\rho_h = 4A_{sh} / (R'_h s_h) \quad (3)$$

where

$$R'_h = R_h - D_h \quad (4)$$

$$R_h = 0.85R \quad (5)$$

$$A_{sh} = 0.25\pi D_h^2 \quad (6)$$

where A_{sh} is the cross sectional area of the spiral bar, R is the diameter of the circular column section, R'_h is the diameter of circle through center of reinforcement , D_h is the diameter of spiral, s_h is pitch of the spiral, R_h is the diameter of circle through outside of reinforcement. The characteristics of the proposed curve in Figure 2 are as follows [15]:

For region AB ($\varepsilon_c \leq \varepsilon_{cc0}$) :

$$\sigma_c = Kf_{cd}\left[(2\varepsilon_c / \varepsilon_{cc0}) - (\varepsilon_c / \varepsilon_{cc0})^2\right] \quad (7a)$$

For region BC ($\varepsilon_{cc0} < \varepsilon_c \leq \varepsilon_{20uc}$):

$$\sigma_c = f_{cd}\left[K - \psi_c(\varepsilon_c - \varepsilon_{cc0})\right] \quad (7b)$$

For region AB' ($\varepsilon_c \leq \varepsilon_{c0}$):

$$\sigma_c = k_3 f_{cd}\left[(2\varepsilon_c / \varepsilon_{c0}) - (\varepsilon_c / \varepsilon_{c0})^2\right] \quad (7c)$$

For region B'C' ($\varepsilon_{c0} < \varepsilon_c \leq \varepsilon_{20u}$):

$$\sigma_c = k_3 f_{cd}\left[1 - \psi(\varepsilon_c - \varepsilon_{c0})\right] \quad (7d)$$

The parameters of the stress-strain behaviour model (Figure 2) are defined below :

$$\varepsilon_{ccu} = K(0.2 / \psi_c + \varepsilon_{c0}) \quad (8)$$

$$\psi_c = \tan\theta_c / f_{cd} = (K - 0.5) / (\varepsilon_{50u} + \varepsilon_{50h} - \varepsilon_{c0}K) \quad (9)$$

$$\psi = \tan\theta / f_{cd} = 0.5 / (\varepsilon_{50u} - \varepsilon_{c0}) \quad (10)$$

where

$$\varepsilon_{50u} = (3 + 0.29k_3 f_{cd}) / (145k_3 f_{cd} - 1000) \quad (11)$$

$$\varepsilon_{50h} = 0.75\rho_h \sqrt{(R_h / s_h)} \quad (12)$$

The strain at the maximum stress, ε_{co} is approximately 0.002 or 0.0022, γ_{mc} is the material coefficient (safety factor) for concrete, k_3 is the ratio of concrete maximum strength to concrete cylinder strength, ε_{ccu} is the concrete strain at the extreme compression fiber of confined concrete,

ε_{cu} is the concrete strain at the extreme compression fiber of unconfined concrete cover [1,3,14, 15].

For any given strain ε_{cm} at the extreme compression fiber and a given concrete stress-strain curve, the compressive stress block parameters k_{1y}, k_{2y}, k_1, k_2 can be determined for unconfined and confined concrete, respectively [9].

2.2. Behaviour Model for Steel

Stress-strain behaviour models for steel are shown in figure 3 including the strain hardening effect for analysis and design [2].

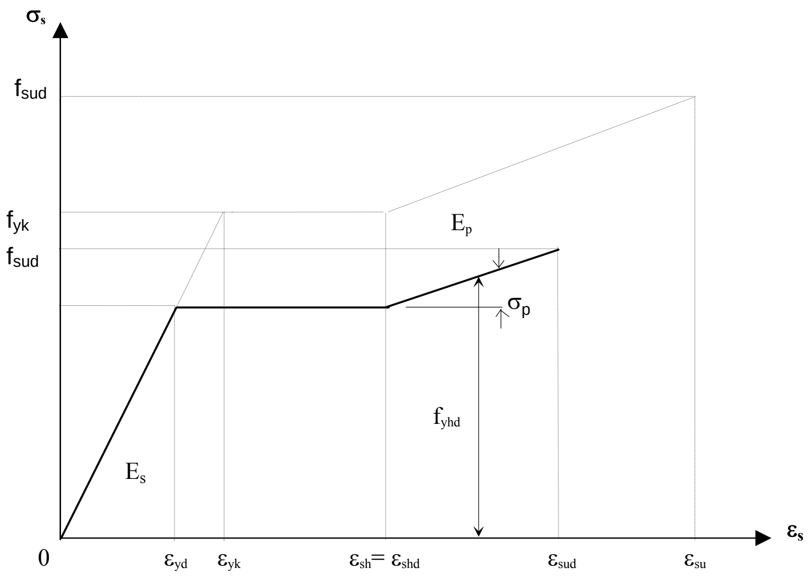


Figure 3. Stress-strain behaviour model for steel, including the effect of strain hardening

In this model, it is assumed trilinear approximate, as in figure 3, considering the upper yield strength and the increase in strain due to strain hardening. The slope of ascending linear portion that is described as plastic behaviour is,

$$E_p = (f_{su} - f_{yk})/(\varepsilon_{su} - \varepsilon_{sh}) \quad (13)$$

where E_p= modulus of plasticity of steel, f_{su}=the failure strength for steel, f_{yk}=the characteristic yield strength for steel, ε_{su} = the ultimate strain for steel, ε_{sh} = the initial value of strain hardening for steel.

The design value of the ultimate strain for steel is,

$$\varepsilon_{sud} = (f_{sud} - f_{yd})/E_p + \varepsilon_{sh} \quad (14)$$

For $\varepsilon_s > \varepsilon_{shd}$ ($\varepsilon_{shd} = \varepsilon_{sh}$) in figure 3, the design value of the upper yield strength is,

$$f_{yhd} = f_{yd} + (\varepsilon_s - \varepsilon_{sh})E_p \le f_{sud} \quad (15)$$

where $f_{sud} = f_{su}/1.3$

2.3. Limit States for Concrete and Steel

Yielding limit for concrete (L_{yc}) can be defined interms of the strain at the maximum stress $\varepsilon_{cy} = \varepsilon_{co}$ or $K\varepsilon_{co}$; ultimate limit (L_{uc}) can be also defined in terms of the strain at the extreme compression fiber of unconfined (ε_{cu}) or confined (ε_{ccu}) concrete. The yielding limit (L_{ys}) and ultimate limit (L_{us}) for steel can be defined with $\varepsilon_{yd} = f_{yd} / E_s$ and ε_{sud}, respectively [2] (Figure 4).

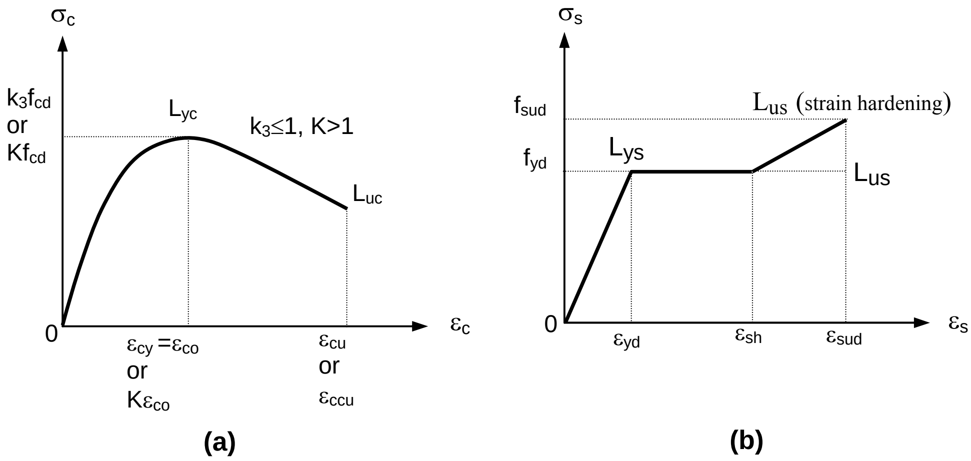


Figure 4. **(a)** Yielding/softening limit L_{yc} and ultimate limit L_{uc} for the concrete **(b)** Yielding limit L_{ys} and ultimate limit L_{us} for the steel

For the columns reaching the ultimate strength at the compression failure zone, the strain will be less than the yielding strain ($\varepsilon_s < \varepsilon_{yd}$). For that reason, the values of N_y, M_y and ϕ_y can be evaluated taking $\varepsilon_{cy} = \varepsilon_{co}$ (0.002 or 0.0022) for the unconfined concrete, $\varepsilon_{cy} = K\varepsilon_{co}$ (0.002 K or 0.0022 K) for the confined concrete. On the other hand, for the columns reaching the ultimate strength at the tension failure zone, criteria for the yielding limit can be described as; for $\varepsilon_s = \varepsilon_{yd}$; $\varepsilon_{cy} < \varepsilon_{co}$ [4].

3. YIELDING ANALYSIS FOR CIRCULAR SHORT COLUMNS

In this part of the study, the algorithms for the N_y, M_y and ϕ_y values for the confined concrete having circular section designed for the ultimate state are derived. The algorithms are given for the certain configuration of the longitudinal reinforcement (for example $n = 10$) as shown in figure 5. The algorithms presented can be easily used with minor alteration for sections having various geometries.

$$r_s = 0.5R - d'$$
$$z_s = 2r_s \cos\alpha$$

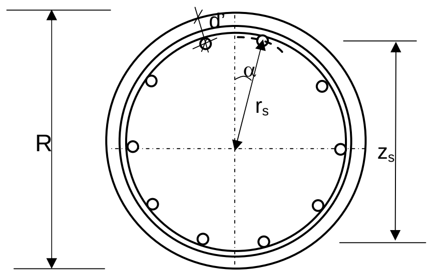


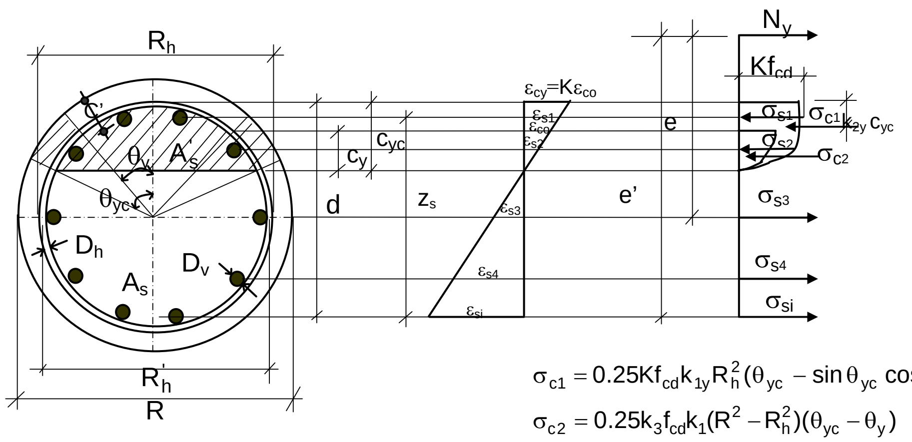
Figure 5. Geometric parameters

3.1. Geometric Parameters

Geometric parameters are shown in figure 5.
For n=10, α can be calculated from the geometry shown in figure 5. In this study, $\alpha = 18^{o}$ and $z_s = 2(0.951)r_s$ are calculated for n=10.

3.2. Yielding Limit Criteria for the Compression Failure

For the columns reaching the utimate strength at the compression failure zone, the values of N_y, M_y and ϕ_y can be evaluated as described in Chap.3 (Fig.6).



$$\sigma_{c1} = 0.25Kf_{cd}k_{1y}R_h^2(\theta_{yc} - \sin\theta_{yc}\cos\theta_{yc})$$
$$\sigma_{c2} = 0.25k_3 f_{cd}k_1(R^2 - R_h^2)(\theta_{yc} - \theta_y)$$

(b1) Cross-section **(b2)** Strain distribution **(b3)** Stress distribution and internal forces

Figure 6. Yielding limit parameters and internal forces for the compression failure

$$\sigma_{cy} = Kf_{cd} \tag{16}$$

In confined sections, the compression region can be defined as the summation of the concrete core confined with circular hoops and unconfined concrete cover. The neutral axis for the concrete cover (c_y) can be written as,

$$c_y = c_{yc} / K \tag{17}$$

where c_{yc} shows that the depth of the neutral axis for the confined core concrete.

The angles θ_y and θ_{yc} for the unconfined and confined concrete can be expressed as [13];

$$\theta_y = \cos^{-1}\left[(0.5R_h - c_{yc} + c_y)/(0.5R_h)\right] \tag{18a}$$

$$\theta_{yc} = \cos^{-1}\left[(0.5R_h - c_{yc})/(0.5R_h)\right] \quad ; \text{for } c_{yc} \le 0.5R_h \tag{18b}$$

and

$$\theta_y = \pi - \cos^{-1}\left[(0.5R_h - c_{yc} + c_y)/(0.5R_h)\right] \tag{19a}$$

$$\theta_{yc} = \pi - \cos^{-1}\left[(c_{yc} - 0.5R_h)/(0.5R_h)\right] \quad ; \text{for } c_{yc} > 0.5R_h \tag{19b}$$

Then, the stresses σ_{si} can be expressed according to strain ε_{si}. The compressive stress block parameters k_{1y} and k_{2y} are calculated [9].

The equilibrium equation obtained from the sum of the internal forces is

$$N_y = \left\{\left[0.25Kf_{cd}k_{1y}R_h^2(\theta_{yc} - \sin\theta_{yc}\cos\theta_{yc})\right] + \left[0.25k_3f_{cd}k_1(R^2 - R_h^2)(\theta_{yc} - \theta_y)\right] + \sum_{i=1}^{n}\sigma_{si}A_{si}\right\} \tag{20}$$

and the equilibrium equation obtained from taking moments about the tension steel is

$$N_y = \left\{0.25Kf_{cd}k_{1y}R_h^2(\theta_{yc} - \sin\theta_{yc}\cos\theta_{yc})\left[0.5(R_h + z_s) - k_{2y}c_{yc}\right] + 0.25k_3f_{cd}k_1(R^2 - R_h^2)(\theta_{yc} - \theta_y)\left[0.5(R_h + z_s) - c_{yc} + c_y - (k_2c_y)\right] + \sum_{i=1}^{n}\sigma_{syi}A_{si}x_i\right\}/(e + 0.5z_s) \tag{21}$$

where N_y is the yielding load.

From these nonlinear equations, N_y and c_{yc} are solved iteratively. In addition M_y and ϕ_y can be derived:

$$M_y = N_y e \tag{22}$$

$$\phi_y = \varepsilon_{cy} / c_{yc} \tag{23}$$

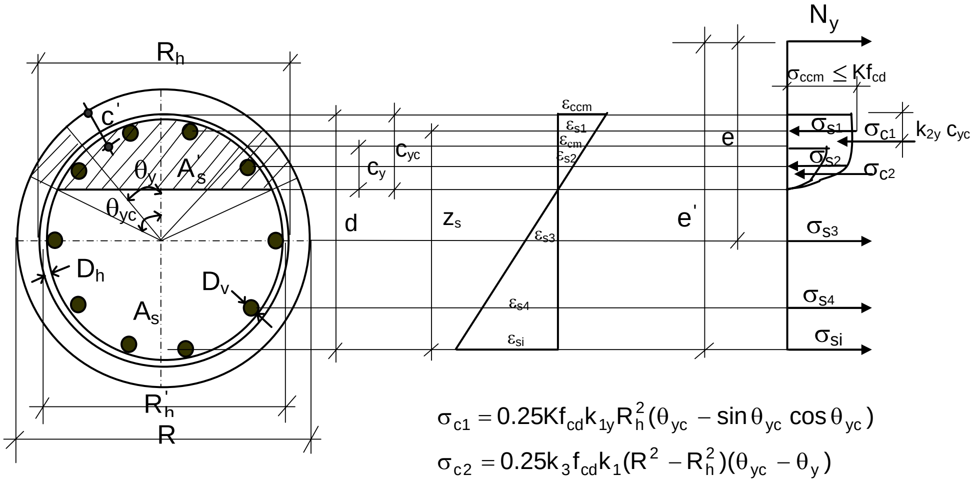


Figure 7. Yielding limit parameters and internal forces for the tension failure

3.3. Yielding Limit for Circular Short Column Which Reaches the Ultimate State by Tension Failure

For a section which reaches the ultimate limit by tension failure, c_y value can be calculated using Eq.19 by assuming c_{yc} value. For $\varepsilon_s = \varepsilon_{yd}$, $\varepsilon_{ccm}(\leq \varepsilon_{cy} = K\varepsilon_{co})$ can be determined by using the equation below (Fig.7):

$$\varepsilon_{ccm} = c_{yc}\varepsilon_{yd} / \left[(0.58R_h + z_s) - c_{yc}\right] \leq \varepsilon_{cy} = K\varepsilon_{co} \quad (24)$$

$\varepsilon_{cm}(\leq \varepsilon_{co})$ value for the unconfined concrete cover can also be written as:

$$\varepsilon_{cm} = \varepsilon_{ccm} c_y / c_{yc} \quad (25)$$

After evaluating the ε_s strains, θ_{yc} and the stresses for the reinforcements can be determined. Depending on the ε_{cm} and ε_{ccm} values, k_1, k_2 and k_{1y}, k_{2y} parameters can also be determined [9]. Substituting these parameters into Eqs.20 and 21, N_y can be evaluated. Until the values of N_y obtained from Eqs.20 and 21 are equal, the depth of neutral axis, c_{yc} is changed. In addition M_y, ϕ_y values for the yielding limit can be evaluated using Eqs.22 and 23 just in the case of the compression failure.

3.4. Numerical Example

In this part of the study, N_y, M_y and ϕ_y values will be evaluated for a column having circular section which is designed according to the ultimate [2].

Input datas:

$N_d = 750\text{ kN}, M_d = 110\text{ kNm}, \ R = 400\text{mm}$

$n = 10$, $f_{ck} = 25\text{MPa}$, $f_{yk} = 220\text{MPa}$, $f_{ywk} = 220\text{MPa}$ $D_h = 10\text{mm}$, $s_h = 100\text{mm}$,

$\gamma_c = 1.5$, $\gamma_s = 1.15$, $E_s = 2 \cdot 10^5\text{ MPa}$, $E_p = 750\text{MPa}$, $\varepsilon_{sud} = 0.114$, $\varepsilon_{sh} = 0.02$,

$\varepsilon_{co} = 0.0022$, $\varepsilon_{cu} = 0.0035$, $k_1 = 0.754, k_2 = 0.443$, $k_3 = 1$, $d' = 50\text{ mm}$

$f_{cd} = f_{ck}/\gamma_c = 16.67\text{MPa}$ $f_{yd} = f_{yk}/\gamma_s = 191.3\text{MPa}$, $\varepsilon_{yd} = f_{yd}/E_s = 0.0009565$,

$e = M_d / N_d = 147\text{ mm}$

$R_h = 0.85 \cdot R = 0.85 * 400 = 340\text{mm}$, $R'_h = R_h - D_h = 330$ mm, $A_{sh} = 0.25\pi D_h^2 = 78.54\text{mm}^2$,

$r_s = 0.5R - d' = 0.5 * 400 - 50 = 150\text{mm}$, $\rho_h = 4A_{sh}/(R'_h \cdot s_h) = 0.00952$,

$\varepsilon_{50h} = 0.75\rho_h(R_h / s_h)^{1/2} = 0.01317$, $K = 1 + (2.05\rho_h f_{yk} / f_{ck}) = 1.17$,

$\varepsilon_{50u} = (3 + 0.29k_3 f_{cd})/(145k_3 f_{cd} - 1000) = 0.005528$,

$\Psi_c = (K - 0.5)/(\varepsilon_{50u} + \varepsilon_{50h} - \varepsilon_{co}K) = 41.56$

$\varepsilon_{ccu} = K(0.2/\Psi_c + \varepsilon_{co}) = 0.0082$, $k_{1c} = 0.827$, $k_{2c} = 0.469$

For the input values above, total area of longitudinal steel in the section is $A_{s_v} = 2366\text{ mm}^2$ (10 ϕ18) and the ultimate curvature is $\phi_{uc} = 0.0495$ rad/m [2].

($e = 147$ mm > $e_b = 52$ mm, therefore, a tension failure occurs.)

Calculate N_y, M_y and ϕ_y for circular column confined by circular spirals.

Solution

Yielding limit state

Try $c_{yc} = 204$ mm

$\varepsilon_{ccm} = c_{yc}\varepsilon_{yd} / [0.5(R_h + z_s) - c_{yc}] = 0.0018$

$c_y = c_{yc} / K = 204.28 / 1.17 = 175$ mm

$\varepsilon_{cm} = \varepsilon_{ccm} c_y / c_{yc} = 0.00154$

$c_{yc} = 204\,\text{mm} > 0.5R_h = 170\,\text{mm}$

$\theta_{yc} = \pi - \cos^{-1}[(c_{yc} - 0.5R_h)/(0.5R_h)] = 1.774$ rad

$\theta_y = \pi - \cos^{-1}[(0.5R_h - c_{yc} + c_y)/(0.5R_h)] = 2.542$ rad

The values σ_{si} may be determined from the strain diagram:

$\varepsilon_{sy1} = [c_{yc} - 0.5(R_h - z_s)]\,\varepsilon_{yd} / [0.5(R_h + z_s) - c_{yc}] = 0.00156$

$\sigma_{sy1} = f_{yd} = 191.3$ N/mm^2

$\varepsilon_{s2} = [c_{yc} - 0.5R_h + 0.588r_s]\varepsilon_{yd} / [0.5(R_h + z_s) - c_{yc}] = 0.00108$

$\sigma_{sy2} = f_{yd} = 191.3$ N/mm^2

$\varepsilon_{sy3} = [c_{yc} - 0.5R_h)]\varepsilon_{yd} / [0.5(R_h + z_s) - c_{yc}] = 0.000303$

$\sigma_{sy3} = 60.6$ N/mm^2

$\varepsilon_{sy4} = [c_{yc} - 0.5R_h - 0.588r_s]\varepsilon_{yd} / [0.5(R_h + z_s) - c_{yc}] = -0.000476$

$\sigma_{sy4} = -95.2$ N/mm^2

$\varepsilon_{sy5} = (-\varepsilon_{yd}) = -0.0009565$

$\sigma_{sy5} = -191.3$ N/mm^2

$k_1 = 0.537$, $k_2 = 0.359$

$k_{1y} = 0.596$, $k_{2y} = 0.365$

$A_{si} = 254.47$ mm^2

$N_y = 666.54$ kN (Eqs.20 and 21)

$M_y = N_y e = 666.54 * 146.67 * (10^{-3}) = 97.76$ kNm

$\phi_y = \varepsilon_{ccm} / c_{yc} = 0.0018 / (204.28 * 10^{-3}) = 0.0088$ rad/m

$\delta = \phi_{uc} / \phi_y = 0.0495 / 0.0088 = 5.6$

4. CONCLUSIONS

Internal forces, bending moments and curvatures for the yielding limit of confined columns having circular sections can be determined easily using the algorithms given in this test. When the concrete reaches the yielding limit of the structural member, it is suggested that the strains in concrete are equal to the strains without any increase in strain. These strains can be accepted for the whole concrete types as 0.002 or 0.0022. Consequently, this acceptance can lead to accurate solution for evaluating of the rotation of the potential plastic hinges and section ductility. In this context, the reliability of the limit and seismic design increase.

5. REFERENCES

[1] ACI Committee 318, (1984a), Building Code Requirements for Reinforced Concrete, ACI Publication, Detroit.

[2] Alacalı, S., Doran, B., Ünal, A., Design for Short Columns with Confined Concrete Having Circular Section Considering Strain Hardening in Reinforcement, Y.T.Ü. Dergisi, Sayı 1, 57-70, Mart 2003.

[3] BSI, (1980), Code of Practice for Structural Use of Concrete (CP 110, Part 1: Design, Materials and Workmanship), British Standards Institution, London.

[4] CEB, “Structural Analysis,” Bulletin d’Information 153, 117-157, Paris, April 1982.

[5] CEB, (1991a), “CEB – FIP Model Code 1990 Final Draft Chapter 1-3”, Bulletin d’Information, 203.

[6] Ersoy, U., (1985b), Betonarme Temel İlkeler ve Taşıma Gücü Hesabı Cilt 1, Bizim Büro Basımevi, Ankara.

[7] Ersoy, U., Tankut, T. ve Uzumeri, S.M., (1987), “The Influence of Strain History and Strain Gradient of Confined Concrete”, Canadian Journal of Civil Engineering, 14 (3).

[8] FIP/CEB, (1990), “High Strength Concrete, State of the Art Report”, Bulletin d’Information, 197.

[9] Gündüz, A., (1986) “Kuşatılmamış Betonlu Dikdörtgen Kiriş Kesitlerinde Moment-eğrilik İlişkilerinin Belirlenmesiyle İlgili Bir Tasarım Algoritması” YTÜ Dergisi, Sayı 1.

[10] Gündüz, A., (1991a), “Yüksek ve Normal Mukavemetli Betonların Davranışının Yanal Donatıyla Yetkinleştirilmesi”, TMMOB İnşaat Mühendisleri Odası İstanbul Şubesi 2. Ulusal Beton Kongresi, 27-30 Mayıs, İstanbul.

[11] Gündüz, A. ve Noyan, S., (1991), “Betonarme kirişlerde plastik mafsal dönme kapasitelerinin belirlenmesiyle ilgili bir yaklaşım”, Yıldız Teknik Üniversitesi Dergisi Sayı 4, İstanbul, 29-36.

[12] Kent, D.C. ve Park, R., (1971), “Flexural Members With Confined Concrete”, Proceedings ASCE, 97 (7).

[13] Mac Gregor, J.G., (1997), Reinforced Concrete Mechanics and Design, Prentice – Hall International Inc., New Jersey.

[14] Park, R., Priestley, M.J. ve Gill, W.D., (1982), “Ductility of Square – Confined Concrete Columns”, Proceedings ASCE, 108 (4).

[15] Park, R., and Paulay, T., (1975), “Reinforced Concrete Structures, Wiley, New York, 769 pp.

[16] Sheikh, S.A., (1982), “A Comperative Study of Confinement Models”,Journal of the American Concrete Institute, 79 (3): 296-305.