ARAŞTIRMA MAKALESİ

A STATISTICAL ANALYSIS FOR THE EQUIVALENT STIFFNESS RATIO OF THE TIE ELEMENTS OF R/C COUPLED SHEAR WALLS

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BETONARME BOŞLUKLU PERDE SİSTEMLERDE BAĞ ELEMANI EŞDEĞER RİJİTLİĞİ İÇİN İSTATİSTİKSEL BİR ANALİZ

ÖZET

Boşluklu perdelerin yapısal analizinde, eşdeğer çerçeve sistemlerin kullanılması oldukça sık başvurulan bir yöntemdir. Yöntemde bağ kirişi rijitliklerinin hesabı oldukça büyük bir önem taşımaktadır. Bu çalışmada, farklı geometriye sahip betonarme boşluklu perde sistemlerin elastik ve plastik bağıntılar kullanılarak sonlu elemanlar tekniği ile çözümleri yapılmış, çok sayıda çözüm sonuçları, istatistiksel veriler olarak dikkate alınmıştır. SPSS(Ver.5.0) paket programı kullanılarak, çok değişkenli regresyon yaklaşımıyla bağ kirişi eşdeğer rijitlik çarpanları elde edilmiştir. Ayrıca elastik ve plastik eşdeğer rijitlik çarpanları arasındaki oranı ifade eden bir formül verilmiştir. Eşdeğer rijitlik çarpanları arasındaki bu oran için en küçük kareler regresyon çizgisi tahmin edilmiştir.

SUMMARY

The use of bar frame modelling has still been one of the current methods in coupled shear wall systems in structural design. In this process, determining the stiffness of the tie-beams has had a great importance. In this study, an adequate number of results obtained by the finite element analysis of the R/C coupled shear wall systems having several geometries in elastic-plastic reigon is considered as a statistical data. Using SPSS(Ver.5.0)-statistical package program, an equivalent tie-beam stiffness modification parameter is provided. It is given a formule which defines the ratio between the plastic and elastic equivalent stiffness modification parameters. For the equivalent stiffness ratio, the least-squares regression line has been estimated.

1. INTRODUCTION

Shear walls are generally used in design of the multi-storey buildings due to their good performance under lateral loads like lateral pressure and/or earthquake inertia forces. Coupled shear walls which are special cases of shear wall systems comprise an effective earthquake resisting structure of high rigidity and reasonable ductility due to their short span tie-beams (Fig.1). In this study, a proposal for the estimation of stiffness of tie-members of coupled shear walls is examined in a plastic reigon. The parameters of the problem are defined and the necessary explanations dealing with the subject are discussed. The stiffness of the coupling members, the geometrical and material parameters that effect the stiffness directly are defined in plastic reigon.

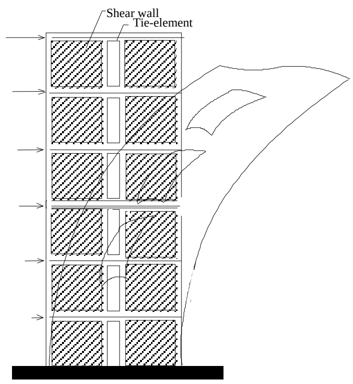

Figure 1. A typical model of coupled shear wall

2. THE NONLINEAR ANALYSIS OF COUPLED SHEAR WALLS BY USING PLASTIC RELATIONS

It is well known that, in the medium and multi-storey buildings, shear walls and coupled shear wall systems are usually used successfully to provide the necessary stiffness, strength and ductility. The two types of modelling are considered in this study for the lateral load analysis of coupled shear wall systems: (1) finite element modelling and (2) the equivalent bar frame modelling. The use of equivalent bar frame modelling for the analysis of the coupled wall system has still been one of the practical methods in design [1, 2, 3, 4, 5]. The structural behavior of coupled shear walls is greatly influenced by the behavior of their coupling beams; therefore the analysis and design of these elements have had a great importance. The geometrical parameters d,b,h,ℓ,L,t of the problem are shown in Fig.2; d, ℓ are the gross height and the clear length of the tie elements, respectively, h is the height of the flat, b is the width of the wall, L is the length measured center to center. The mechanical parameters of the problem are E,G,μ which are modulus of elasticity, shear modulus and the Poisson's ratio, respectively [4].

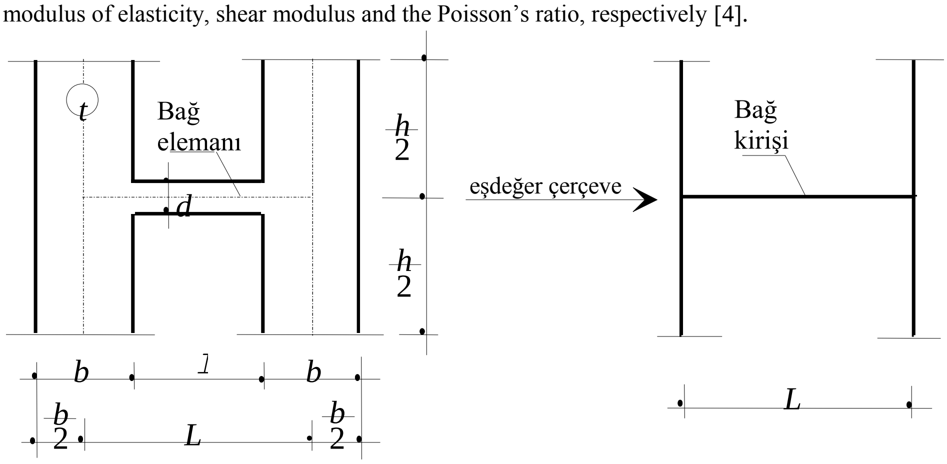

Figure 2. Coupled shear wall and equivalent frame [4]

The behavior of the tie elements, which connect two shear walls, depends on the geometry of the tie elements and the mechanical characteristics of the concrete and reinforcement [2, 4]. To estimate the real behavior of the coupled shear wall systems, it is necessary to consider the system in plastic reigon. In this study, to produce sample data of statistical assessment, coupled wall systems are analyzed under the classical plasticity concepts by the LUSAS program working with the NLFEA (Nonlinear Finite Element Analysis) techniques. Since it is a quasi-brittle material, the nonlinear behavior of concrete can be modeled using the concepts of classical plasticity theory. It is essential to choose a suitable yield criterion in order to analyze any member using classical plasticity concepts [6, 7] The analytical model of Drucker-Prager yield criterion which is a smooth approximation to the Mohr-Coulomb theory is used to model the nonlinear behavior of the concrete [4].

A smooth approximation to the Mohr-Coulomb surface was expressed by Drucker and Prager in the following form [8]:

$$f(I_1, J_2) = \alpha I_1 + \sqrt{J_2} - k = 0 \tag{1}$$

where I_1, J_2 are the first invariant of the stress tensor and second invariant of the stress deviator tensor, respectively, α and k are positive constants pertaining to the material. α and k are related to Mohr-Coulomb constants c (cohesion) and ϕ (friction) by

$$\alpha = \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)}, \; k = \frac{6c\cos\phi}{\sqrt{3}(3-\sin\phi)} \tag{2}$$

These two parameters which define the strength of the material are used by the LUSAS program in the plastic analysis. The most suitable values for the parameters c=2.80 -3.70 MPa and ϕ =25°-35° are selected [4, 6, 7, 9].

The nonlinear behavior of reinforcement can also be modelled using the concepts of classical plasticity theory. The analytical model of von-Mises yield criterion which states that yielding begins when the octahedral shearing stress reaches a critical value k is used to model the nonlinear behaviour of the reinforcement [6, 7]. The yield surface was expressed by von-Mises in the following form [8]:

$$f(J_2) = J_2 - k^2 = 0 \tag{3}$$

where k is the yield stress in pure shear.

In this study only the configuration of the reinforcement for the tie elements is taken into account (Fig.3). The reinforcement of the shear wall is not the dominant character for the nonlinear behavior of the global system [4]. Thus, this effect has been eliminated in this part of the study.

The coupled shear wall systems are analyzed using 2D elements for the geometrical parameters h =3 m, L =6 m, b=3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6 (m), d =0.20, 0.40, 0.60, 0.80, 1.00, 1.20 (m). The stiffnesses of the tie elements are evaluated as

$$m_{i\theta_i} = \frac{M_i}{\theta_i} \tag{4}$$

where M_i and θ_i are the bending moment and the rotation for section i, respectively (Fig.2). On the otherhand it is known that the stiffness that contains the shear effect of the tie element of the equivalent frame is

$$\overline{m}_{i\theta_i} = \frac{6EI}{L}\frac{L^2}{(L^2+3.9d^2)} \tag{5}$$

where EI is the bending stiffness. L and d are the geometrical parameters shown in Fig.2. Thus, the stiffness of the coupling members (ij member), the geometrical parameters and the material parameters that effect to the stiffness directly are defined in plastic reigon:

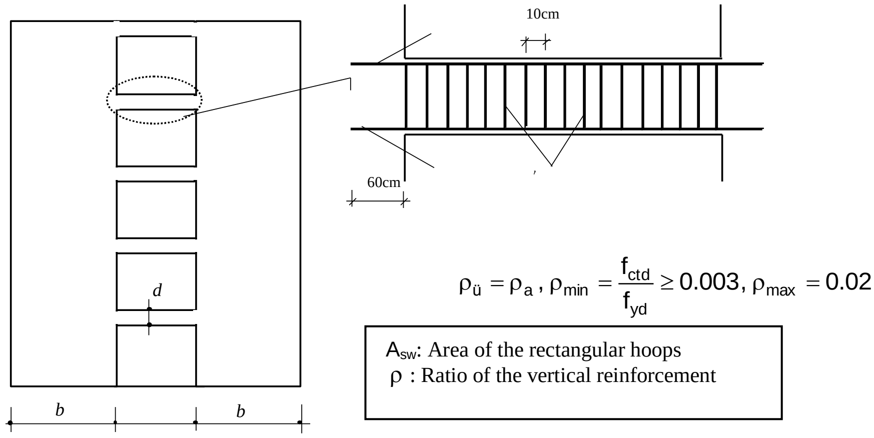

Figure 3. Single span coupled shear wall and the configuration of the reinforcement [4]

$$m_{i\theta_j} = \eta^p \overline{m}_{i\theta_j} \tag{6}$$

where η^p is the stiffness modification parameter. By the way, modification parameter can also be provided as a function below [4]**:**

$$\eta^p = a_o \left(\frac{h}{\ell}\right)^{a_1} \left(\frac{b}{\ell}\right)^{a_2} \left(\frac{d}{\ell}\right)^{a_3} \left(\frac{\sigma_c}{f_c}\right)^{a_4} \tag{7}$$

The adequate number of results which are obtained by finite element analysis in plastic reigon is considered as a statistical sample data and then using SPSS package program, the constants a_o, a_1, a_2, a_3, a_4 in Eq.7, can be provided [4]:

$$\eta^p = 1.507 \left(\frac{h}{\ell}\right)^{0.0281} \left(\frac{b}{\ell}\right)^{1.6896} \left(\frac{d}{\ell}\right)^{-0.5124} \left(\frac{\sigma_c}{f_c}\right)^{-0.345} \tag{8}$$

or by rounding the powers,

$$\eta^p = 1.5 \left(\frac{h}{\ell}\right)^{0.03} \left(\frac{b}{\ell}\right)^{1.70} \left(\frac{d}{\ell}\right)^{-0.51} \left(\frac{\sigma_c}{f_c}\right)^{-0.35} \tag{9}$$

where, σ_c / f_c (stress in exciting zones/characteristic compresive strenght for the concrete) defines the various stress level which will be suggested by the researchers. For example considering $\frac{\sigma_c}{f_c} \cong 0.40$, the behaviour of the system can be idealized as linear [4]. Examples made by using the produced formulae are clearly proved that the obtained procedure is accurate enough. The numerical examples and the comparisons made with some available data in literature and with the results produced by finite element analysis, show that the provided formula which take into account the principal geometrical and mechanical parameters, is quietly satisfactory. Also the stiffness of the coupling members and the geometrical and material parameters that effect to the stiffness directly are defined in linear-elastic reigon [4, 5]. The stiffness of the ij member is:

$$m_{i\theta_j} = \eta^e \overline{m}_{i\theta_j} \tag{10}$$

İn a similar way, the stiffness modification parameter in elastic reigon is provided [4, 5]:

$$\eta^e = 1.9210\left(\frac{h}{\ell}\right)^{0.0282}\left(\frac{b}{\ell}\right)^{1.6824}\left(\frac{d}{\ell}\right)^{-0.5860} \qquad (11)$$

or by rounding the powers

$$\eta^e = 1.9\left(\frac{h}{\ell}\right)^{0.03}\left(\frac{b}{\ell}\right)^{1.70}\left(\frac{d}{\ell}\right)^{-0.60} \qquad (12)$$

To comment this study much easier, dividing Eq.8 to Eq.11;

$$\frac{\eta^p}{\eta^e} = 0.80 \cdot \left(\frac{d}{\ell}\right)^{0.09}\left(\frac{\sigma_c}{f_c}\right)^{-0.35} \qquad (13)$$

is provided. As shown in Fig.4, for higher stress levels, values of $\frac{\eta^p}{\eta^e}$ decrease rapidly. And if the numerical datas are pointed in E3 space, it is possibble to obtain a lineer relation between $\frac{\eta^p}{\eta^e}$ and $\frac{\sigma_c}{f_c}$ (Fig.4).

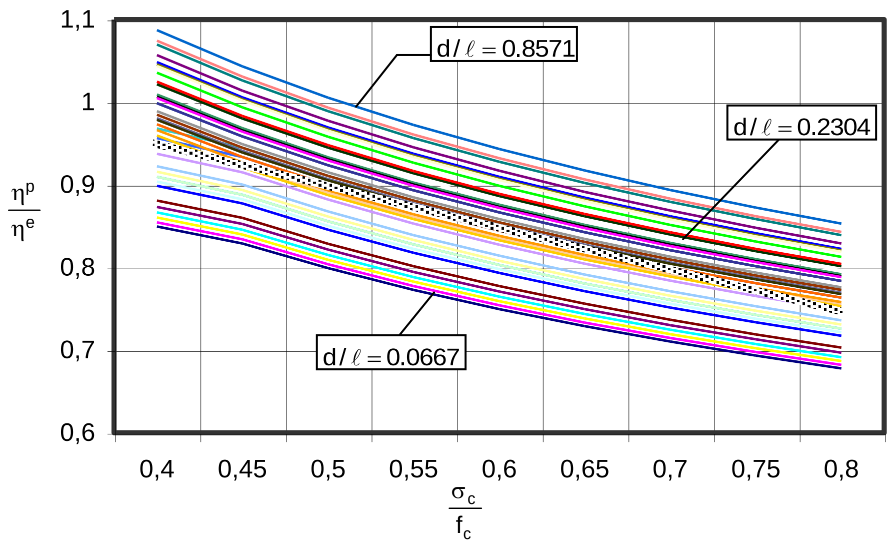

Figure 4. Experimentally obtained ($\frac{\eta^p}{\eta^e}$ - $\frac{\sigma_c}{f_c}$) curves for a different values of d/ℓ [4]

3. REGRESSION WITH CONSTANT VARIANCE

The conditional density function of the random variable *Y*, for constant value X = x is $f(y|x)$ [10, 11, 12]. $Y|x_i$ for $X = x_i$ shows random variable Y_i. Supposing this relationship is linear, it may be written

$$E(Y|X=x) \equiv m_{Y|x} = \alpha + \beta x \tag{14}$$

where E and m are expected and mean values respectively. α and β are constants and the variance of Y may be independent or a function of x. This is known as the linear regression of Y on X. This relationship is represented by $y = \alpha + \beta x$. The values of parameters α and β are estimated with sample data. These estimations are shown as $\hat{\alpha}$ ve $\hat{\beta}$. Also, the estimation of the regression line is $\hat{y} = \hat{\alpha} + \hat{\beta}x$. Conceivable, there could be many straight lines, depending on the values of α and β, that might qualify as the mean-value function of y in the light of the data. The best line may be the one that passes through the data points with the least error. Therefore the line with the least total error can be obtained by minimizing the sum of the squared errors-that is, by minimizing

$$\Delta^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}(y_i - \hat{\alpha} - \hat{\beta}x_i)^2 \tag{15}$$

to obtain α and β, where n is the number of data points. From which the least-squares estimates, the $\hat{\alpha}$ and $\hat{\beta}$ are provided as follows:

$$\hat{\alpha} = (1/n)\sum y_i - (\hat{\beta}/n)\sum x_i = \bar{y} - \hat{\beta}\bar{x} \tag{16}$$

$$\hat{\beta} = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} \tag{17}$$

$\sum = \sum_{i=1}^{n}$. n is the sample size, $\bar{x}, \bar{y}$ are the sample means.

As a result, the least-squares regression line is

$$\hat{y} = \hat{\alpha} + \hat{\beta}x \tag{18}$$

Since the general trend is accounted for through the regression line of the Eq.18, the variance about this line is the measure of dispersion of interest, which is the conditional variance $Var(Y|x)$ [10, 11, 12]. For the case where the conditional variance $Var(Y|x)$ is assumed to be constant within the range of x of interest, an unbiased estimate of this variance is

$$s_{Y|x}^2 = \frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \frac{1}{n-2}\left[\sum_{i=1}^{n}(y_i - \bar{y})^2 - \hat{\beta}^2\sum_{i=1}^{n}(x_i - \bar{x})^2\right] = \Delta^2/n-2 \tag{19}$$

Thus the corresponding conditional standard deviation is $s_{Y|x}$, the coefficients $\hat{\alpha}$ and $\hat{\beta}$, and $s_{Y|x}^2$, are the estimates of the respective true values of α, β and $Var(Y|x)$.

Confidence Intervals

The exact confidence interval for the $\alpha + \beta x$ can be estimated using Eq.20 [10, 11]:

$$(\hat{\alpha} + \hat{\beta}x_k) \pm t_{\alpha/2,n-2}\, s_{Y|x}\left[\frac{1}{n} + \frac{(x_k - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right]^{1/2} \tag{20}$$

In the present case, of course, $t_{\alpha/2,n-2}$ (f, degree of freedom) denotes the precentile value of the t-distribution with $n-2$ degrees of freedom. $t_{\alpha/2,f}$ is the value of the the value T at the cumulative probability $(1-\alpha/2)$, as shown in tables [10, 11, 12].

Prediction Intervals

The prediction interval for value Y can be estimated for x_k value of x [11]

$$(\hat{\alpha}+\hat{\beta}x_k) \pm t_{\alpha/2,n-2}s_{Y|x}\left[1+\frac{1}{n}+\frac{(x_k-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right]^{1/2} \tag{21}$$

The smallest of confidence and prediction intervals are observed for $x=\bar{x}$.

4. STATISTICAL ANALYSIS OF THE EQUIVALENT STIFFNESS RATIO

In this part of the study, the analysis of statistical relation between the independent variable ($x = \sigma_c / f_c$) and dependent variable ($y = \eta^p / \eta^e$) is aimed, for different values of (d/ℓ). The values η^p / η^e corresponding to a given value σ_c / f_c are obtained by Eq.13.

All the sample datas (Table 1) are selected from ref. [4]. When pairwise data for two variables, say X and Y, are plotted on a two dimensional graph, such as shown in Fig.5, It shows that this relationship is likely linear. After this assumption, it can be estimated the least-squares regression line, and the 95 % confidence and prediction intervals of Y for σ_c / f_c = 0.40.

Table 1. $x = \sigma_c / f_c$ and $y = \eta^p / \eta^e$ variables for d/ℓ =0.2304

i	1	2	3	4	5	6	7	8	9
x_i	0.40	0.45	0.5	0.55	0.60	0.65	0.70	0.75	0.80
y_i	0.966	0.927	0.893	0.864	0.838	0.815	0.794	0.775	0.758

$\sum_{i=1}^{9} x_i y_i = 4.501,\ \sum_{i=1}^{n} x_i^2 = 3.39,\ \bar{x} = 0.60,\ \bar{y} = 0.85$

$$\hat{\beta} = \left[\frac{4.501 - 9(0.60)(0.85)}{3.39 - 9(0.60)^2}\right] = -0.592$$

The estimated regression line is:

$\hat{y} = 1.205 - 0.592x$.

The value estimated for $x = 0.40$ is

$\hat{y} = 1.205 - 0.592(0.40) = 0.968$

Confidence interval: Before, the variance of regression line should be determined:

$\hat{y}_i = 1.205 - 0.592x_i$

$$\Delta^2 = \sum_{i=1}^{9}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}(y_i - \hat{\alpha} - \hat{\beta}x_i)^2 = 17.32 \cdot (10^{-4})$$

$s_{Y|x}^2 = 17.32(10^{-4})/7 = 0.000247,\ s_{Y|x} = 0.01573$

According to Eq.20 the 95 % **confidence interval** for $x = 0.40$ is

$1-\alpha = 0.95,\ \alpha/2 = 0.025.$

With $p = 1-\alpha/2 = 1-0.025 = 0.975$, and $f = n-2 = 9-2 = 7$; $t_{0.025,7} = 2.365$

from Table EC5 . [11]:

$$1.205 - 0.592(0.40) \pm (2.365)(0.01573)\left[(1/9) + (0.40 - 0.60)^2 / 0.15)\right]^{1/2} = (0.945\,;\,0.991)$$

Prediction interval: According to Eq.21, the 95 % prediction interval for $x = 0.4$ is

$$1.205 - 0.592(0.40) \pm (2.365)(0.01573)\left[1 + (1/9) + (0.40 - 0.60)^2 / 0.15)\right]^{1/2}$$
$$= (0.925\,;\,1.012)$$

At the same time, for $d/\ell = 0.0667$;

$\hat{y} = 1.071 - 0.518x$, $s_{Y|x} = 0.01284$, and the 95 % **confidence and prediction intervals**, for $x = 0.40$ are $(0.845\,;\,0.882)$ and $(0.828\,;\,0.899)$, respectively.

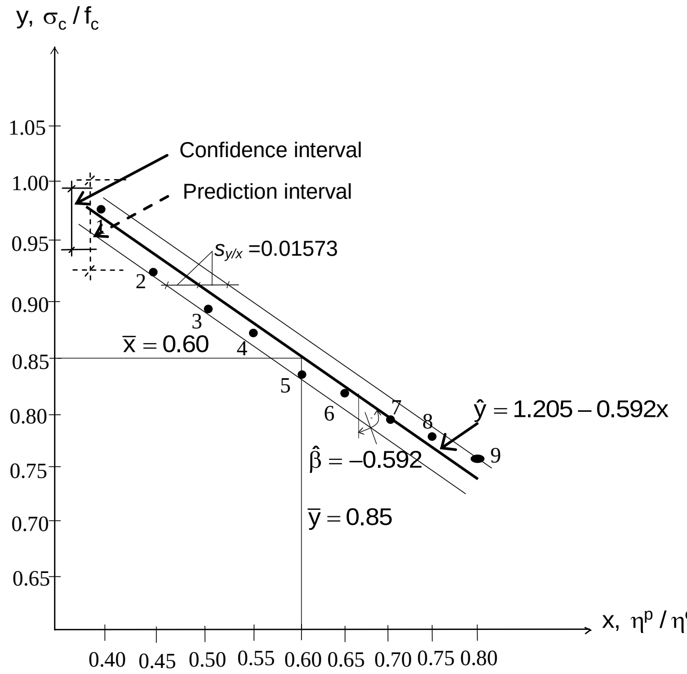

Figure 5. Regression line, confidence and prediction interval for η^p / η^e

5. CONCLUDING REMARKS

In region which has $0.40 \le \sigma_c / f_c \le 0.80$, Eq.13 can be expressed as a linear function. In this context, the analysis of statistical relation between the independent variable σ_c / f_c and dependent variable η^p / η^e , for different values of (d/ℓ) has been examined. The least-squares regression line, and the 95 % confidence and prediction intervals of η^p / η^e have been estimated. Thus, the least-squares regression line can be assumed for Eq.13.

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