ARAŞTIRMA MAKALESİ

POWER DISTRIBUTION SYSTEM EXPANSION PROBLEM

Belgin TÜRKAY

Faculty of Electrical & Electronics, Istanbul Technical University, Electrical Engineering Department, Maslak-ISTANBUL

Geliş Tarihi: 11.06.2001

DAĞITIM ŞEBEKELERİNDE BÜYÜME PROBLEMİ

ÖZET

Genel olarak, herhangi bir sistemin büyümesinde dikkat edilmesi gereken en önemli konu, zaman içerisinde artan yükün ekonomik, güvenilir mümkün olduğu kadar,güvenli bir şekilde karşılanmasıdır. Doğal olarak, güvenlik ve güvenilirlik diğer sistemlerden farklı olarak dağıtım sistemlerinde işletmeden kaynaklanan ayarlamalar ve sınırlamalar getirmektedir. Çalışmada karma değişkenli programlama yöntemi kullanılmıştır.

ABSTRACT

In general, the primary goal in any system expansion is to timely meet the growth of demand in the most economical, reliable, and safe manner possible. Of course, safety and reliability introduce certain operational regulations and constraints that are different in distribution systems than other systems, and therefore must be considered in the expansion plans. In This paper, mixed integer programming method is used for formulation.

Keywords: Power Distribution planning, distribution expansion, mathematical programming

1. INTRODUCTION

Various plans to timely meet this demand growth, are continually studied for all major components of the electrical systems namely, generation, transmission, and finally distribution.

In so far as the distribution system expansion is concerned, optimisation methods used may be divided in to two distinct categories.

i) Mathematical programming methods

ii) Heuristic methods, including expert systems and evolutionary algorithms

i) *Mathematical Programming Algorithms*

Numerical optimisation techniques such as linear programming in general have been proven to converge to the global optimum as opposed to just an acceptable answer as noted form [1-4]. On the other hand, the mathematical complexities for modelling and decomposition of the real life problems to fit the mathematical models, as well as computational difficulties, make these techniques very challenging.

Mathematical optimisation techniques may be divided into continuous and discrete types. Although the continuous techniques have far less computational complexity, the distribution expansion problem is inherently discrete. For this reason, the overwhelming majority of the past research has utilized the discrete optimisation techniques.

ii) Heuristic methods:

Evolutionary techniques in general, are stochastic searches modelled similar to some of the well-known natural or evolutionary concepts. In Genetic Algorithms (GA) for example, a solution to a problem evolves from an initial point, similar to the evolution of the DNA strings. Various decisions based on a given design criteria, are usually modelled as a binary string where each position on the string represents a particular decision. Several bits in the string [1] represent decisions involving more than two choices. The algorithm begins with a user provided or randomly generated population of candidate solutions. A feasibility evaluation is performed for each individual in the population. From those individuals performing best, the algorithm performs crossover (combining the strings of two parents) and mutation (random changes in the status of one or more bits in the string). The process continues its search and is stopped by a predetermined stopping rule while concluding that; no better solution is likely to be found [1]. It should be realized that, in general, there is no guarantee that GA will find a global optimal solution.

The distribution expansion is a nonlinear, NP-Complete practical problem, with a 40-year history of continued efforts and contributions for improved solutions [5-19].

The developed computer program minimises the objective function, which is defined as the total cost of distribution network by determining the optima of

i) distribution transformer locations and power,
ii) load transfers between the demand centers,
ii) line routes and their sizes,
ii) load flow in the lines,
ii) and power of the transformers,

subject to a set of design constraints.

The computational results that obtained by implementing the model onto an example model are found to be satisfactory.

2. DESIGN CRITERIA AND ASSUMPTIONS

To aid formulation, note the following definitions and assumptions

1) Candidate Substation locations, all or a subset of which are to be developed, are assumed known (Fig. 1).
2) Location and loading data for existing and future load centers are available, and the load is distributed on main laterals known as local loops (Fig. 2).
3) All local loops are self contained in their protective and power factor correction equipment and viewed as a unity power factor load at the inter connection point to main feeder.
4) Any feeder link between two nodes may have a multiple routing options, and multiple size options may consider for each routing (Fig.3).
5) Unit costs for every option, and the characteristic data for all equipment are known.

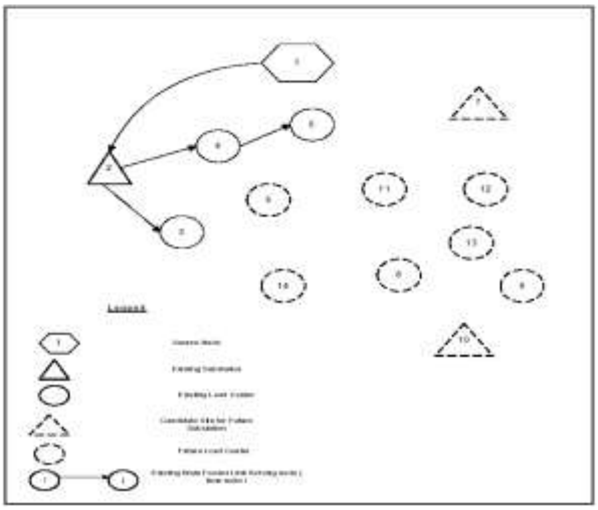

Figure1. Existing and future substations and load centers

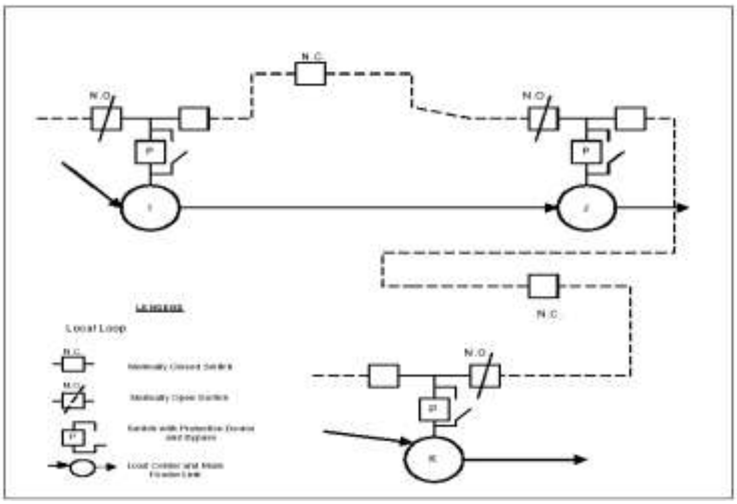

Figure 2. Local loops supplied by nodes I and j

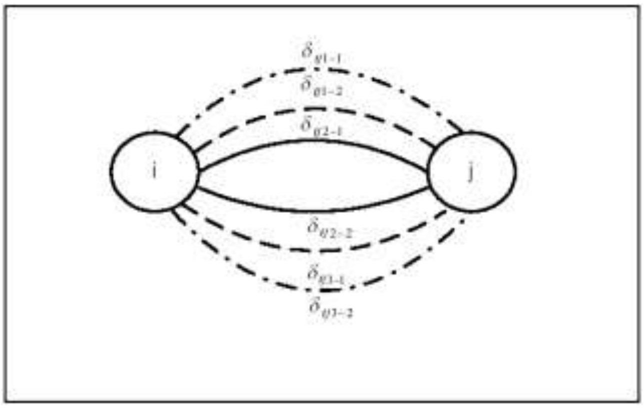

Figure3. Routing and size options

Fixed Costs

Fixed cost, or the zero order cost as defied by [20], refers to a one-time expenditure for installation of any equipment. It contains the cost of material, transportation, and labor for the installation and commissioning of the facility. Fixed costs are independent of loading and therefore should not be modelled as functions of power flows. Fixed costs are also the dominant costs of the expansion project and are usually paid in instalments over the future years.

Variable Costs

Variable Costs in general refers to costs that are functions of loading such as the costs associated with production and transport of energy. Although the operating costs are generally considered as variable costs, not all operating costs are functions of loading. For those that may be modelled as functions of loading, they differ radically as functions of types of loading. Many researchers, such as [17-19] have modelled the operating and maintenance (O&M) costs as linear or quadratic functions of loading without distinction to load type. Before expressing the types of variable costs that should be considered, it is important to discuss budget and the standard engineering economic studies that are normally performed during planning analyses.

Optimisation vs. Engineering Economic analysis

An optimisation study, with the same objective (as the engineering economics study), on the other hand, seeks the plan with minimum cost among all possible plans. There is no equivalency restriction for this optimisation algorithm. As long as the stated constraints are satisfied, the plan is considered a feasible one. The feasible plan with minimum cost is the solution regardless of its equivalency to other feasible plans. Another fundamental difference is that in an engineering economic study, all constraints are assumed satisfied. Based on the foregoing argument, in contrast to [12], we suggest that the levelized carrying charges should not be used in an optimisation study in the same manner as used in an engineering economic study.

Instead, the *inflation adjusted present worth costs* will be used for both fixed and variable costs calculated based on [21] using the following formula

$$P:W:C_{(n)} = \left[\frac{1+i_f}{1+i} \right]^n \quad (1)$$

where

P.W.C $_{(n)}$: is the inflation adjusted present worth cost of the facility installed in year *n*

C : is the current cost of the facility

n : is the number of years to installation of the facility with current cost *C*

i_f : is the inflation rate (assumed 5% in our study)

i : is the fixed charge rate (assumed 14% in our study)

The validity of this approach comes form the fact that the treatment is the same for every expenditure incurred in every stage. It should be noted that in general, designers are always searching for alternatives that differ more of the capital costs to the future, as these are usually the more economical plans. Similarly, the foregoing approach in cost modelling inherently searches for the same alternatives. It is further suggested that the only variable cost that need be considered is the total cost of energy losses in the distribution system alone. This is because all other O&M costs (load type dependent or otherwise), similar to fuel, production, and transport costs, are common among all alternatives. Excluding all other costs except the energy losses, the variable costs become very small compared to the fixed costs.

3. MODELLING AND FORMULATION

Mixed integer programming formulation is proposed for this problem. Objective function

$$\min C = \sum_{t=1}^{n} \left\{ C_{f,t}^T \, \delta_t + \frac{1}{2} \left[X_t^T Q X_t \right] \right\} \quad (2)$$

where

$C_{f,t} \in R^n$:is the vector of fixed costs in stage *t*

$\delta_{S,t} \in [0,1]^n$:is the vector of decisions at stage *t*

Note further with a diagonal Q, a completely decoupled expansion of (19) is

$$\min C = \sum_{t=1}^{T} \left\{ \sum_{ij \in Lposs} C_{f,ij,t} \, \delta_{ij,t} + \sum_{ij \in Lpos} C_{v,ij,t} \, X_{ij,t}^2 \right\} \quad (3)$$

where

$C_{f,ij,t}$: is the fixed cost coefficient of link ij at stage t.

$C_{v,ij,t}$: is the variable cost coefficient of link ij at stage t.

L_{poss} : is the set of all link possibilities

$X_{ij,t}$: is the diversified power flow in the link ij at stage t

4. PROBLEM DEFINITION

The primary goal of the expansion problem is to timely serve the load growth safely, reliably, and economically. Here, it is assumed that safety considerations have already been translated into a set of operational standards in the design stage. Reliability and economics on the other hand, may be formulated as objectives for optimisation programs.

First, a single criterion optimisation is developed to minimize the total fixed and variable costs at all stages ensuring that;

- every demand center *j* is served for all stages,
- voltages are within guidelines at every node *j* for all stages,
- all elements operate within their capabilities and operational constraints,
- all expenditure is within the budget for every stage.

A general mathematical representation of the above is

$$\min C = \sum_{t=1}^{T} \left\{ \sum_{S \in stations} Cf_{S,t} + \sum_{S \in sations} Cv_{S,t} + \sum_{F \in feeders} Cf_{S,t} + \sum_{F \in feeders} Cv_{S,t} \right\} \tag{4}$$

Subject to

$$\sum X_{ij,t} = \sum X_{jk,t} = P_{j,t} \quad \forall\, j \in Load\ Centers, ij\ and\ jk \in Feeders \tag{5}$$

$$V^{\min} \le V_{j,t} \le V^{\max} \qquad \forall\, j \in Load\ Centers \tag{6}$$

$$S_{i,t} \le S_i^{\max} \qquad \forall\, i \in Stations \tag{7a}$$

$$X_{ij,t} \le X_{ij,t}^{\max} \qquad \forall\, ij\ Feeder\ Links \tag{7b}$$

$$\sum_{S \in stations} Cf_{S,t} + \sum_{S \in sations} Cv_{S,t} + \sum_{F \in feeders} Cf_{S,t} + \sum_{F \in feeders} Cv_{S,t} \le B_t, \forall\, t = 1,2,...,T \tag{8}$$

where

T : is the number of stages to full expansion
t : is each stage of the stage process
$X_{ij,t}$: is the directional complex power flow from node i j at stage t
$X_{jk,t}$: the directional is complex power flow from node j at stage t to node k
P^{jet} : peak demand of the diversified is load centre (node) j at stage
$C_{fs,t}$: is the fixed cost of substation S to be installed at stage t
$C_{vS,t}$: is the variable cost of substation S to be incurred at stage t
$C_{fF,t}$: is the fixed cost of feeder F to be installed at stage t,
$C_{vF,t}$: is, the variable cost of to be incurred at stage t feeder F
$V_{j,t}$: is the voltage at node j at stage t
V^{min}, V^{max} : are the lower &upper bounds of acceptable voltage

$S_{i,t}$, S_i^{Max} : are loading of and maximum at stage t substation S Capability respectively
$X_{ij,t}$, $X_{ij,t}$: are the flow in at stage t the link ij and Maximum Capability respectively

The value of *C* to be minimized in equation (4) is the total cost for the expansion of the system over all the stages. Constraints (5) through (8) include both physical and performance conditions. Constraint (5) is the well known Kirchhoff's Current Law (KCL) applied to every node. This is also known as the flow conservation law in mathematical literature. If there is no local demand at the node, it is usually referred to as the transhipment node. Constraint (6) sets explicit voltage limits for all the load centres. Constraints (7a) and (7b) ensure that all substation transformers and feeders are loaded within their capabilities, and all other operational conditions are within limits. Finally, constraint (8) is a budgetary constraint so that the expansion costs at each stage are within the budgeted amount. As discussed previously, although this is an important constraint to include in all practical planning, it has been generally neglected in all previous formulations.

For calculation purposes, the present worth/unit length costs multiplied by the length for each option was used to find fixed costs $C_{f,ij,t}$ For the variable cost coefficients $C_{v,ij,t}$ the present total cost of energy was used based on the following formula.

$$C_{v,ij,t} = \frac{(C_{e,t})(r_{ij})(l_{ij})(LLF)(8760)(10^3)}{KV^2_{LL}} \tag{9}$$

where

$C_{e,t}$: is the present value of the total cost of energy incurred at stage *t.*
r_{ij} : is the resistance of the conductor in ohms/mile for the link *ij*
l_{ij} : is the length of the conductor for the link *ij* in km
LLf : is the loss load factor (assumed 15%)
KVLL : is the feeder Line-Line operating voltage in KV
8760 : is the number of hours at stage *t,* (one year)

Introducing the necessary decision variables, separating the linear and the non-linear terms, and assuming for now (this will be shown later) that all variable costs may be modelled as quadratic functions of power flows, a matrix form representation of the problem may be formulated as shown below.

$$\min C = \sum_{t=1}^{T} \left\{ C^T{}_{fs,t}\ \delta_{F,t} + C^T{}_{fF,t} + \frac{1}{2}\left[X^T_{S,t}\ Q_F\, X_{F,S} + X^T_{F,f}\, Q_F\, X_{F,t} \right] \right\} \tag{10}$$

$$A_j X_t = P_{j,t} \qquad X_t = \left[X_{S,t}\ X_{F,t} \right]^T \tag{11}$$

$$V^{\max} \le V_{j,t} \le V^{\max} \tag{12}$$

$$X_t \le b_t \tag{13}$$

$$C^{T}_{fS,t}\ \delta_{F,t} + \frac{1}{2}\left[X^{T}_{S,t} Q_S X_{S,t} + X^{T}_{F,t} Q_F X_{F,t}\right] \leq B_t \qquad \forall t \qquad (14)$$

where

$C \in R$:is the total cost for the ultimate system expansion
$t \in Z$:is the stage number of the multistage study
$m \in Z$: is the total number of nodes
$n \in Z$: is the total number of feeders and the substations
$n_S \in Z$: is the number of Substations
$B_t \in Z$: is the expansion budget for stage *t*
$X_{F,t} \in R^{(n+nS)}$: is the vector of feeder power flows
$X_{S,t} \in R^{nS}$: is the vector of substation loads
$X_t \in R^n$: is the feeder/substation loading vector
$\delta_{F,t} \in [0,1]^{(n+nS)}$: is the vector of feeder decision variables
$\delta_{S,t} \in [0,1]^{nS}$: is the vector of substation decision variables
$Vj,t \in R^m$: is the vector of node voltages
$Cfs,t \in R^{nS}$: are the vectors of fixed substation costs
$P_{j,t} \in R^m$: is the vector of Load Center demands
$CfF,t \in R^{(n+nS)}$: is the vectors of fixed feeder costs

The above problem is a nonlinear and mixed integer optimisation problem. Mixed integer problems in general, and specifically this problem, computationally belong to the class NP complete. NP completeness (as opposed to Polynomial Bounded ness or class *P*) refers to a class of problems for which algorithmically, the computational complexity of the solution searches grows exponentially (non polynomially) with some parameter [8]. That is, NP complete problems have a computational complexity of the highest degree, and are difficult problems to solve.

6. NUMERICAL EXAMPLE

A simple test case of Fig.4 consisting of one substation and two load-centers, was studied base on the foregoing formulation. Data on routing / size options, and other characteristic data for the system links have been given in table1. The demand data are shown in table 2. Unit costs for fixed and variable expenditures have been provided in table 3 [20]. Several, 10 year, three stage. The optimisation code is developed according to the cutting plane algorithm, which is one of the algorithms of mixed integer programming method.

In all studies, two and eight year periods were considered between the first two stages, and a 30-year period for cumulative loss calculation beyond stage 3. Further, uniform load growths were considered for the first two stages, and it was assumed that the system is fully developed beyond stage 3 with no load growth.

Table 1. Physical and characteristic data for possible links [20].

From		To	r	*l*
1	-	2	0.2	1
1	-	2	0.4	1
2	-	3	0.0982	5
2	-	3	0.1019	7

2	-	4	0.0966	4
2	-	4	0.1019	6
3	-	4	0.1468	6
3	-	4	0.0966	6
3	-	4	0.142	6.7
3	-	4	0.0982	6.7
3	-	4	0.1457	7.5
3	-	4	0.1019	7.5

Table 2. Demand data [20]

Load center	Stage 1 (MW)	Stage 2 (MW)	Stage 3 (MW)
3	6	8	10
4	0	5	8

Table 3. Fixed and variable costs for links

From		To	Var. Cost ($/MVA)	Fixed Cost ($ 10E6)
1	-	2	15	0
1	-	2	15	0.84
2	-	3	3	0
2	-	3	4	6.8376
2	-	4	2	0.887
2	-	4	4	5.8608
3	-	4	3	1.2672
3	-	4	3	1.3306
3	-	4	3	1.8396
3	-	4	3	1.9103
3	-	4	4	7.128
3	-	4	4	7.326

Figure 4. Test system configuration

Table 4. Optimisation Solutions

	From	To	Select Option	Flow (MW)	Volts
Stage 1	1	2	1-1	6	126
	2	3	1-1	6	125.1
	2	4			
	3	4			
Stage 2	1	2	1-1	13	126
	2	3	1-1	8	124.8
	2	4	1-1	5	125.4
	3	4			
Stage 3	1	2	1-2	18	126
	2	3	1-1	10	124.5
	2	4	1-1	8	125.04
	3	4			

In all studies, two and eight year periods were considered between the first two stages, and a 30-year period for cumulative loss calculation beyond stage 3. Further, uniform load growths were considered for the first two stages, and it was assumed that the system is fully developed beyond stage 3 with no load growth.

7. CONCLUSIONS

A general mathematical model is developed for the optimisation of distribution networks, which can be also implemented to low voltage networks. The model is coded using mixed integer programming method. The optimisation program minimises the total cost of low voltage distribution network as the objective function by determining the optima of the number, locations and powers of distribution transformers, the routes of the lines and the voltage drops and power losses within the network subject to a set of constraints.

A categorical analysis of the past 40 years of research indicates that even though many advances have been made towards the solution of the distribution expansion problem, there still remain many areas for future research. Implementation of multiple routing and size options, inclusion of upgrade possibilities, and treatment of reliability and other objectives are major areas in need of future development regardless of the techniques. A mixed integer programming modelling in a three-stage formulation including multiple routing and size options has been implemented. It has been shown that inclusion of multiple size gradation is a significant factor when implementing explicit voltage constraints. Investigations of a simple test case indicate that inclusion of voltage constraints without consideration of multiple routing and size options render the consideration of multiple routing and size options render the solution sub optimal. It is also proposed that upgrade possibilities need be considered for the problem to be practical. This is vital for maximum asset utilization, optimality of the solution, and it is inherently aligned with industry practices and training.

The only variable costs that can influence the solution and need be considered are the energy losses. Although neglecting the losses or piecewise / stepwise linearization techniques can provide adequate solutions, inclusion allows for more accurate modelling. It is proposed that reliability, social/environmental impacts, and other objectives be considered as separate objectives and not integrated into single objective formulations.

REFERENCES

[1] H. L. WILLIS, H. TRAM, M. V. Engel, and L. Finley, “Selecting and Applying Distribution Optimisation Methods”, IEEE Computer Applications in Power, pp. 12-17, January 1996.

[2] F. CHOOBINEH, T. BURGMAN, “Transmission Line Route Selection: An Application of K-Shortest Paths and Goal Programming”, IEEE Transactions on Power Apparatus and Systems, Vol. 103, No. 11, pp. 3253-3259 November 1984,

[3] T. H. FAWZI, et al., “Routing Optimisations of Primary Rural Distribution Feeders”, IEEE Transactions on Power Systems, Vol. 101, No. 5, pp. 1129-1133, May 1982.

[4] K. M. SHEBI, S. M. IBRAHIM, “Approximate Technique For The Optimal Route Of Distribution Feeders”, IEEE, Region 10, New York, pp. 1172-1177.
1987.

[5] KNIGHT, U.G.W.,” The Logical Design of Electrical Networks Using Linear Programming Methods”, Proc. IEE, Vol. 107 A, pp. 306-316, 1960.

[6] OLDFIELD, J.V.and LANG, M.A., “Dynamic Programming Network Flow Procedure for Distribution System Planning”, Proceeding Power Industry Computer Applications Conference, 1965.

[7] ADAMS, R.N. and LAUGHTON, M.A., “Optimal Planning of Power Networks Using Mixed Integer Programming”, Proc. IEEE, Vol.121, No.2, pp. 139-147, February 1974.

[8] CRAWFORD, D.M. and HOLT, S.B., "A Mathematical Optimisation Technique for Locating and Sizing Distribution Substations and Deriving Their Optimal Service Areas", IEEE Transactions on Power Apparatus and Systems, Vol. PAS. 94, No. 2, pp. 230-235, April 1975.

[9] MASUD, E.,"An Iterative Procedure for Sizing and Timing Distribution Substations Using Optimisation Techniques", IEEE PES Winter Meeting, pp. 1281-1286, February 1974.

[10] GÖNEN, T.and FOOTE, B.L.," Distribution System Planning Using Mixed-Integer Programming", Proc. IEE, Vol. 128, Pt.C., No. 2, pp. 70-79, March 1981.

[11] CARSON, M.J.and CORNFIELD, G., "Design of Low-Voltage Distribution Networks", Proc IEEE, Vol. 120, No. 5, pp. 585-593, May 1973.

[12] CARSON, M,J. and, CORNFIELD, G., "Computer Aided Design of Low-Voltage Distribution Networks", Computer Aided Design, IEE Conf. Publ. 86, pp. 121-124, 1972.

[13] HINDI, K.S., "Design of Low-Voltage Distribution Networks, A Mathematical Programming Method", Proc. IEE, Vol. 124, No. 1, pp. 54-58, January 1977.

[14] HINDI, K.S., BRAMMELLER, "A., Optimal Cable Profile L.V. Radial Distributors", Proc. IEE, 123, No. 4, pp. 331-334, April 1976.

[15] PONNAVAIKKO, M., RAO, P.," Distribution System Planning Through A Quadratic Mixed Integer Programming Approach" IEEE Transaction on Power Delivery, Vol. PWRD-2, No.4, October 1987.

[16] HSU, Y., CHEN, J.," Distribution Planning Using a Knowledge-Based Expert System" " IEEE Transaction on Power Delivery, Vol.5, No.3, pp. 1514-1519 July 1990.

[17] JONNAVITHULA, S., BILLINTON, R., "Minimum Cost Analysis of Feeder Routing in Distribution System Planning", IEEE Transaction on Power Delivery, Vol.11, No.4, pp. 1935-1940, October 1996.

[18] M.A.EL-Kady, " Computer Aided Planning of Distribution Substation and Primary Feeders" IEEE Transaction on Power Apparatus and Systems", Vol.103, No.6,pp.1183-1189 June 1984.

[19] BALANCHARD, M., Delorme, L., Simard, C. and Nadeau " Experience with Optimisation Software for Distribution Planning" IEEE Transaction on Power Systems, Vol.11, No.4, pp. 1891-1898, November 1996.

[20] Y:TANG " Power Distribution System Planning with reliability Modelling and Optimisation" IEEE Transactions on Power System, Vol. 11, No.1, , pp.181-189, February 1996