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**ARAŞTIRMA MAKALESİ**

# THE INFLUENCE OF THE RHEOLOGICAL PARAMETERS ON THE THEORETICAL STRENGTH LIMIT IN COMPRESSION OF VISCOELASTIC UNIDIRECTIONAL COMPOSITE MATERIALS

**ARZU ÇİLLİ**

*Yıldız Technical University, Faculty of Arts and Sciences, Dept. of Physics, Topkapı-İSTANBUL*

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## TEK YÖNLÜ VİSKOELASTİK KOMPOZİT MALZEMELERİN KURAMSAL DAYANIKLILIK SINIRINA REOLOJİK PARAMETRELERİN ETKİSİ

### ÖZET

Bugüne değin, sıkışma durumundaki tek yönlü kompozit malzemelerin dayanıklılık sınırı, parça parça bakıldığında sabit özellikler gösteren bu malzemelerin normalleşmiş özellikler taşıyan, yapısal olarak türdeş ortotropik ortamlar aracılığıyla modellenmesinde kullanılan sürekli yaklaşım çerçevesinde incelemişti [3,2].
Yukarıda sözü edilen problemler üzerinde çalışırken, kritik gerilme durumunun türdeş olduğuna ve incelenen kompozitlerin mekanik özelliklerinin zamandan bağımsız olduğuna dikkat etmek gerekir.
[4] numaralı yayında sözü edilen konuyu dikkate alarak, doğrusal, tek yönlü, viskoelastik kompozit malzemelerin sıkışma durumundaki kuramsal dayanıklılık sınırının incelenmesinde kullanılan yaklaşım önerilmiştir. Bu durumda incelenen kompozit malzemenin yapısında, birbirini güçlendiren katmanların ya da liflerin birtakım yerel eğrilikleri olduğu varsayılmıştır.
[4] numaralı yayında önerilen yöntemi kullanarak kompozit malzemeleri oluşturan bileşenlerin reolojik parametrelerinin kritik zaman (yani, kuramsal dayanıklılık sınırı) üzerindeki etkisi incelenmiştir.

### SUMMARY

Up to present the theoretical strength limit in compression of unidirectional composite materials has been studied in the framework of the continual approach according to which these materials with piecewise-constant properties are modelled by a structurally homogeneous ortohotropic medium with normalized characteristics [3] and [2].
Note that under studying of the above problems it is assumed that the precritical stress-strain state is homogeneous and the mechanical properties of the considered composites are time-independent. Taking into account the above-stated in paper [4], the approach for the investigations of the theoretical strength limit in compression for the linear viscoelastic unidirectional composite materials is proposed. In this case it is assumed that in the structure of the considered composite material there are some local curvings of the reinforcing layers or fibers. Using the method proposed in the present paper [4], the influence of the rheological parameters of the components of the composite materials on the critical time (i.e. on the theoretical strength limit) is investigated.

## 1. INTRODUCTION

In the investigations carried out with the use of the continuum approach the unidirectional composite materials with piecewise-constant properties are modelled by a structurally homogeneous orthotropic medium with normalized properties. Based on this consideration and according to [3] and [2] the type change of the equations TDLTS written for the this homogeneous orthotropic infinite body under acting at infinity of the external compressive forces is studied. In these cases, if the condition of ellipticity of the TDLTS equations is not satisfied and these equations lose their ellipticity, then it is assumed that the fracture of the considered material occurs. Moreover, in these cases the values of the external forces corresponding to the above type change of the equations TDLTS are acccepted as the theoretical strength limit in compression of the considered unidirectional composites.
Note that, while studying the above problems, it is assumed that the precritical stress-strain state is homogeneous and the mechanical properties of the considered composites is time-independent. Taking into account this stuation, in the paper [4] with concrete problems as an example, the approach for the investigations of the theoretical strength limit of the viscoelastic composite materials has been proposed.
In the present paper using the method proposed in [4] the influence of the rheological parameters of the components of composite materials to the critical time is investigated.

## 2. FORMULATION OF THE PROBLEM, METHOD OF SOLUTION AND SELECTION OF THE FRACTURE CRITERIA

For the simplicity of the consideration we consider a composite material consisting of the alternating layers of two materials with insignificantly local curving in the structure (Figure 1) under compression along the reinforcing layers at "infinity" by uniformly distributed normal forces of intensity p.

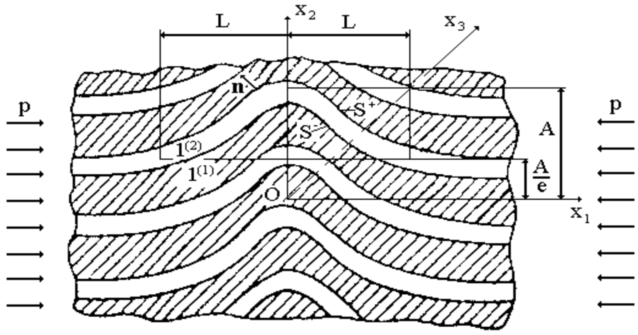


**Figure 1.** The initial imperfection with the local curving form in the structure of the considered composite material.

The reinforcing layers will be assumed to be located in planes which are parallel to the plane $ox_1x_3$ and the thickness of every filler layer will be assumed constant. Values related to the matrix will be denoted by upper indices (1); values related to the filler, by upper indices (2). The material of layers of filler is assumed to be only elastic with mechanical characteristics $E^{(2)}$ (Young's modulus) and $\nu^{(2)}$ (Poisson coefficient).

The material of the layers of the matrix is assumed to be linearly viscoelastic with operators

$$E^{(1)} = E_0^{(1)}\left[1-\omega_0\Pi_\alpha^*(-\omega_0-\omega_\infty)\right]$$
$$\nu^{(1)} = \nu_0^{(1)}\left[1+\frac{1-2\nu_0^{(1)}}{2\nu_0^{(1)}}\omega_0\Pi_\alpha^*(-\omega_0-\omega_\infty)\right] \tag{1}$$

where $E_0^{(1)}$ and $\nu_0^{(1)}$ are momentary values of the Young's modulus and of the Poisson coefficient, respectively; $\omega_0$ , $\omega_\infty$ and $\alpha$ are rheological parameters of the matrix material; $\Pi_\alpha^*$ is the fractional-exponential operator of [1] and this operator is determined as

$$\Pi_\alpha^*(\beta) = \int_0^\infty \Pi_\alpha(\beta, t-\tau)\, d\tau\ , \tag{2}$$

where

$$\Pi_\alpha(\beta,t) = t^\alpha \sum_{n=0}^{\infty} \frac{\beta^n t^{n(1+\alpha)}}{\Gamma\left[(n+1)(1+\alpha)\right]} \tag{3}$$

In Eq.(3), $\Gamma(x)$ is the Gamma function. Moreover, in writing Eq.(1), the volumetric expansion and the compression of the matrix are assumed to be elastic only.

Taking into account the periodicity of the composite structure shown in Figure 1 in the direction of the $ox_2$ axis with period $2(H^{(2)} + H^{(1)})$, (where $2H^{(2)}$ is a thickness of the filler, $2H^{(1)}$ is a thickness of the matrix layer), among the layers considered we single out two of them, i.e. $1^{(1)}$, $1^{(2)}$ (Figure 1) and discuss them below.

We associate the corresponding Lagrangian coordinates $o^{(k)}x_1^{(k)}x_2^{(k)}x_3^{(k)}$ ( k=1,2; $x_1^{(1)} = x_1^{(2)} = x_1$ ; $x_3^{(1)} = x_3^{(2)} = x_3$ ) which in their natural state coincide with Cartesian coordinates and are obtained from $ox_1x_2x_3$ by parallel transfer along $ox_2$ axis, with the middle surface of each layer of the filler and matrix.

We assume that the above insignificantly local curving of the reinforcing layers, there are only in the direction of $ox_1$ axis and we investigate the plane deformation state in the considered composite in framework of the piecewise-homogeneous body model with the use of the exact equations of the geometrically non-linear theory of viscoelasticity.

The initial insignificant local curvings of the filler layers are given through the equation of the middle surface of $1^{(2)}$th filler layer as $x_2^{(2)} = \varepsilon f(x_1)$ . Here $\varepsilon$ is a dimensionless small parameter ($0<\varepsilon<<1$) the geometric meaning of which is described by the specifically prescribed form of the function $f(x_1)$. Moreover, it is supposed that the function f and its first derivative is continuous and satisfy the conditions $f(x_1) \to 0$ ; $df/dx_1 \to 0$ at $|x_1| \to \infty$ . Thus, with the above-stated the formulation of the considered problem is exhausted.

For investigations of the fracture of the considered composites in compression, after determination of the stress-deformation state in those with the use of the solution procedure described in [3] it is necessary to select the failure criteria. According to [4] and others we also assume that the exhaustion of the load-carrying capacity of the considered unidirectional composites with local curving in the structure occurs as the result of the stability loss in the material structure. However, in our investigation the stability loss in the structure of the considered composite material is determined as the case where the values of the initially insignificant local curving rise significantly or become infinite under considerable finite values of the external compressive forces.

**3. NUMERICAL RESULTS**

We introduce the dimensionless rheological parameter $\omega = \omega_\infty / \omega_0$ and the dimensionless time $t' = \omega_0^{1/(1+\alpha)} t$ and assume that p does not depend on time t. Before the discussions of the critical time obtained for the investigated problem, we consider some principal moments of the results obtained for the corresponding pure elastic problem. Note that, in this case the critical values of p are determined from the requirements $\left| u_2^{(2),1}(0,0) \right| / L \to \infty$ under $p \to p_{cr.}$. Analysis of numerous numerical results obtained for $p_{cr.}$ shows that the values of $p_{cr}$ do not depend on $\chi = H^{(2)}/L$ and on the local curving form in the structure of the considered composite material. Moreover, by direct verification it is proved that the values of $p_{cr}$ coincide with those obtained in the framework of continual approach proposed in [3] and [2]. Consequently the values of $p_{cr.}$ determined in the framework of the approach [4] are the theoretical strength limit in compression of the considered pure elastic composite material.

Consider now the influence of $E^{(2)}/E_0^{(1)}$ and of the rheological parameters $\omega$ and $\alpha$ on the values of $t'_{cr.0}$ and $t'_{cr.\infty}$. In Table 1, the values of $t'_{cr.0}$ and $t'_{cr.\infty}$ are given for various $E^{(2)}/E_0^{(1)}$. These numerical results indicate that with increasing $E^{(2)}/E_0^{(1)}$ the values of TSLC ( i.e. $t'_{cr.0}$ and $t'_{cr.\infty}$) decrease. The influence of the change of the rheological parameter $\omega$ on the values of $t'_{cr.0}$ and $t'_{cr.\infty}$ are given in Table 2. It follows from the numerical results given in Table 2 that, with the growing $\omega$ (i.e. with decreasing ($E_0^{(1)} - E_\infty^{(1)}$ ), where $E_\infty^{(1)}$ is the long time modulus of the matrix material), the values of $t'_{cr.0}$ and $t'_{cr.\infty}$ increase monotonously.

**Table 1.** The values of $t'_{cr.0}$ and $t'_{cr.\infty}$ for various $E^{(2)}/E_0^{(1)}$ under $p/E_0^{(1)}$ =0.28, $\alpha$ =-0.3, $\omega$=0.5, $\eta^{(2)}$=0.2.

| $E^{(2)}/E_0^{(1)}$ | $t'_{cr.0}$ | $t'_{cr.\infty}$ |
|---|---|---|
| 50 | 0.4900 | 0.4799 |
| 100 | 0.4699 | 0.4650 |
| 150 | 0.4499 | 0.4499 |
| 200 | 0.4000 | 0.4000 |

**Table 2.** The values of $t'_{cr.0}$ and $t'_{cr.\infty}$ for various $\omega$ under $\eta^{(2)}$=0.2, $p/E_0^{(1)}$ =0.42, $E^{(2)}/E_0^{(1)}$ =100, $\alpha$ =-0.3.

| $\omega$ | $t'_{cr.0}$ | $t'_{cr.\infty}$ |
|---|---|---|
| 0.5 | 0.0350 | 0.0350 |
| 1.0 | 0.0390 | 0.0390 |
| 2.0 | 0.0544 | 0.0549 |
| 3.0 | 0.0649 | 0.0649 |
| 5.0 | 0.1890 | 0.1890 |
| 6.0 | 0.6490 | 0.6490 |

Moreover, these numerical results show that $t'_{cr.\infty} < t'_{cr.} < t'_{cr.0}$ ($t'_{cr.0}$ , $t'_{cr.\infty}$ are obtained at $t'_* = \infty$ , $t'_* = 0$ , respectively) and the distinction between $t'_{cr.\infty}$ and $t'_{cr.0}$ (i.e. the values of $t'_{cr.0} - t'_{cr.\infty}$) significantly less than the values of $t'_{cr.0}$ or $t'_{cr.\infty}$. Note that the determination of $t'_*$ is given in [4] and [5].

Consequently, the obtained numerical results can be taken as the theoretical strength limit of the considered viscoelastic composite material, with very high accuracy.

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