ARAŞTIRMA MAKALESİ

EDGE NEIGHBOUR INTEGRITY OF TOTAL GRAPHS

Aysun OZAN AYTAÇ*, Vecdi AYTAÇ**

**Department of Math. and Comp. Science of Science Faculty, Ege University, Bornova – İZMİR*
***Computer Engineering Department, Ege University, Bornova – İZMİR*

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TOTAL GRAFLARIN AYRIT KOMŞU BÜTÜNLÜĞÜ

ÖZET

G = (V, E) bir graf olsun. G grafından bir e ayrıtına bağlı u ve v tepeleri çıkarıldığı zaman G deki e = [u, v] ayrıtı bozulma olarak adlandırılır. Eğer S = {e_1, e_2, …, e_m} deki ayrıtların herbiri G den çıkarılmışsa bu durumda S ayrıtlar kümesine G nin ayrıt bozulma stratejisi adı verilir. S, G grafının bir ayrıt bozulma stratejisi varken, geriye kalan alt graf G/S olsun. Bir G grafının ayrıt komşu bütünlüğü ENI(G) ile gösterilir ve aşağıdaki gibi tanımlanır;

$$ENI(T(P_n)) = \min_{S \subseteq E(G)} \{|S| + \omega(G/S)\}.$$

Burada S, G grafının herhangi bir ayrıt bozulma stratejisi ve $\omega(G/S)$, G/S nin en büyük bileşenin tepe sayısıdır. Bu çalışma total grafların ayrıt komşu bütünlüğü üzerine bazı sonuçları içerir.
Anahtar Kelime: Graf Teorisi, Bağlayıcılık, Ayrıt Komşu Bütünlüğü

SUMMARY

Let G = (V, E) be a graph. An edge e = [u, v] in G is said to be *subverted* when the incident vertices, u, v, of the edge e are deleted from G. A set of edges S = {e_1, e_2, …, e_m} is called an *edge subversion strategy* of G if each of the edges in S has been subverted from G. Let G/S be the survival-subgraph left when S has been an edge subversion strategy of G. The edge–neighbour-integrity of a graph G, ENI(G), is defined to be ENI(T(P_n)) = $\min_{S \subseteq E(G)} \{|S| + \omega(G/S)\}$, where S is any edge subversion strategy of G, and $\omega(G/S)$ is the maximum order of the components of G / S. This paper includes some results on the edge-neighbour-integrity of total graphs.
Keywords: Graph theory, Connectivity, Edge-Neighbour-Integrity

1. INTRODUCTION

Barefoot, Etringer and Swart introduced integrity and edge-integrity as a measure of the vulnerability of graphs to disruption caused by the removal of vertices or edges. [1, 2] Goddard and Swart investigated further the bounds and properties of the integrity of graphs.[9]

A network can be modeled by a graph whose vertices represent the stations and whose edges represent the lines of communication. The vulnerability of the communication network, measures the resistance of the network to disruption of operation after the failure of certain stations or communication links. Many graph - theoretic parameters have been used to describe the stability of the communication network, including connectivity, integrity, and tenacity. We consider that two graphs have same connectivity; but the orders of their largest components are not equal. Then, these two graphs must be different with respect to stability. How can we measure that property? This idea offers the concept of integrity that is different from connectivity.

A graph G, it is denoted by G=(V(G), E(G)), where V(G) is the set of vertices of G and E(G) is the set of edges of G. The number of vertices and the number of edges of the graph G are denoted by |V(G)|, |E(G)| respectively. In a graph, considering the model of a network topology [12], efficiency decreases when some vertices or edges are destroyed in any way. The connectivity of a graph G is the minimum number of vertices whose removal from G results in a disconnected or trivial graph, it is denoted by k(G) [11]. The connectivity of a graph equals the minimum number of vertices whose deletion from the graph disconnects it into two or more components. But, for two graphs that have same connectivity (vertex or edge) and have same number of vertices, the number of vertices of their largest component after any disruption, can be different. Thus, connectivity is not enough for the measure of stability. In graph theory besides connectivity, many graph parameters have been used to describe the stability of graphs, including connectivity, integrity, neighbor-integrity, toughness, edge-integrity, edge neighbor-integrity and tenacity [3, 4, 5, 7, 8]. In this paper we first give the edge-neighbour-integrity as a measure of the stability of a graph. Afterwards we give some results on the edge-neighbour-integrity of total graphs

Let G = (V, E) be a graph. An edge e = [u, v] in G is said to be *subverted* when the incident vertices, u, v, of the edge e are deleted from G. A set of edges S = {e_1, e_2, …, e_m} is called an *edge subversion strategy* of G if each of the edges in S has been subverted from G. let G/S be the survival-subgraph left when S has been an edge subversion strategy of G. The edge-neighbour-integrity of a graph G, ENI(G), is defined to be

$$\mathrm{ENI}(T(P_n)) = \min_{S \subseteq E(G)} \{|S| + \omega(G/S)\}$$

where S is any edge subversion strategy of G, and $\omega(G/S)$ is the maximum order of the components of G / S. [4, 6] This paper includes results on the edge-neighbour-integrity of total graphs.

$\lceil x \rceil$ is the smallest integer greater than or equal to x. $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

2. BASIC RESULTS ON EDGE NEIGHBOUR INTEGRITY

In this section we will review some of the known results.

Theorem 2. 1 : [6] The edge neighbour integrity of

(a) the complete graph K_n is

$$ENI(K_n) = \lceil n/2 \rceil.$$

(b) the path P_n is

$$ENI(P_n) = \lceil 2\sqrt{n+2} \rceil - 3.$$

(c) the star $S_{1,\ n-1}$, where $n \geq 3$, is

$$ENI(S_{1,\ n-1}) = 2.$$

Definition 2. 1: The vertices and edges of a graph are called its elements. Two elements of a graph are neighbors if they are either incident or adjacent. The **total graph** T(G) of the graph G = (V(G) , E(G)), has vertex set V(G) $\cup$ E(G), and two vertices of T(G) are adjacent whenever they are neighbors in G. It is easy to see that T(G) always contains both G and the Line graph L(G) as induced sub graphs. The total graph is the largest graph that is formed by the adjacent relations of elements of a graph [10].

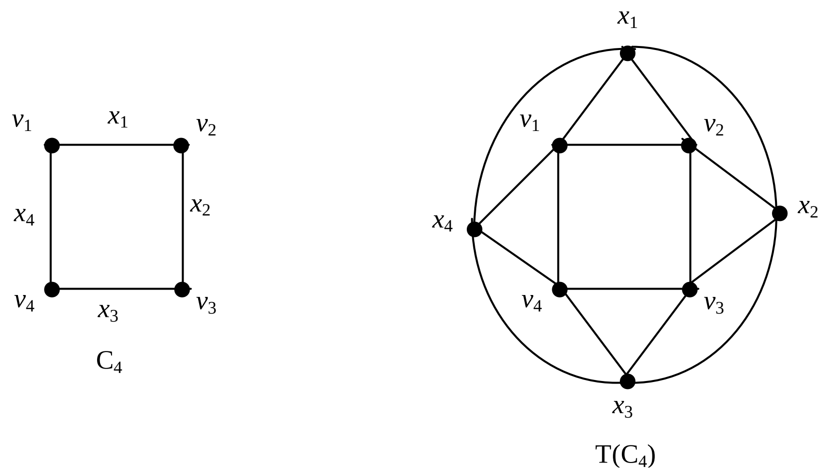

Figure 1. Formation of a total graph.

3. EDGE NEIGHBOUR INTEGRITY OF SOME TOTAL GRAPHS

In this section we give the results on edge neighbour integrity of some total graphs and its some compounds as theorems.

Theorem 3. 1: Let P_n be an n-vertex path. Then,

$$ENI(T(P_n)) = \lceil 2\sqrt{2n+1} \rceil - 3.$$

Proof: The number of vertices of the total graph $T(P_n)$ is 2n–1. Let $V(T(P_n)) = \{v_1, v_2, ..., v_{2n-1}\}$ and S be any subset of $E(T(P_n))$. The subversion of an edge $e = [v_i, v_{i+1}]$ from $T(P_n)$ is the removal of the vertices v_i and v_{i+1} from $T(P_n)$, so $w(T(P_n) - S) \geq \frac{2n-1-2|S|}{|S|+1}$.

Let | S | = m,

$$ENI(T(P_n)) = \min_{S \subseteq E(T(P_n))} \{|S| + \omega((T(P_n))/S)\}\}$$

Then, the function is $f(m)=m+\frac{2n-1-2m}{m+1}$, for m>0. From $f'(m)=\frac{m^2+4m-2n}{(m+1)^2}=0$ and $m = -1 + \sqrt{1+2n}$, and the minimum value of f(m) is $f(m) = -3 + 2\sqrt{1+2n}$.

Consequently, $ENI(T(P_n)) = \lceil 2\sqrt{1+2n} \rceil - 3$.

This completes the proof.

Theorem 3. 2 : Let C_n be cycle of n vertices. Then,

$$ENI(T(C_n)) = \lceil 2\sqrt{2n} \rceil - 2.$$

Proof: The number of vertices of the total graph $T(C_n)$ is 2n. In this case function f(m) is $f(m) = m + \frac{2n-2m}{m}$. Similar to proof of Theorem 3.1, the minimum value of f (m) is $f(m) = 2\sqrt{2n} - 2$ and $ENI(T(C_n)) = \lceil 2\sqrt{2n} \rceil - 2$.

Theorem 3. 3: Let $S_{1,n}$ be a star of n vertices. Then,

$$ENI(T(S_{1,n})) > \lceil \sqrt{4n+2} \rceil - 1, \text{ for } n \geq 10 .$$

Proof: The number of vertices of the total graph $T(S_{1,n})$ is 2n+1. In this case function f(m) is $f(m) = m + \frac{2n+1-2m}{2m}$. Similarly to proof of Theorem 3.1, the minimum value of f (m) is $f(m) = \sqrt{4n+2} - 1$ and $ENI(T(S_{1,n})) > \lceil \sqrt{4n+2} \rceil - 1$, for $n \geq 10$. For n<10s given the following table:

Table 1. $ENI(T(S_{1,n}))$, for n<10

n	3	4	5	6	7	8	9
ENI	3	4	4	5	5	5	6

Definition 3. 1: The (Cartesian) product G1 x G_2 of graphs G_1 and G_2 has $V(G_1) \times V(G_2)$ as its vertex set and (u_1, u_2) is adjacent to (v_1, v_2) if either $u_1 = v_1$ and u_2 is adjacent to v_2 or $u_2 = v_2$ and u_1 is adjacent to v_1.

Next we give the edge neighbour integrity of total graphs of $K_2 \times P_n$ and $K_2 \times C_n$.

Theorem 3. 4: Let $K_2 \times P_n$ be with 2n vertices mesh. Then,

$$ENI(T(K_2 \times P_n)) \geq \lceil \sqrt{6(5n-2)} \rceil - 3.$$

Proof: The number of vertices of the total graph $T(K_2 \times P_n)$ is 5n – 2. In this case function f(m) is $f(m) = m + \dfrac{5n-2-2m}{\frac{2m}{3}}$. Similarly to proof of Theorem 3.1, the minimum value of f (m) is $f(m) = \sqrt{6(5n-2)} - 3$ and $ENI(T(K_2 \times P_n)) \geq \lceil \sqrt{6(5n-2)} \rceil - 3$, for n ≥5. The following table is given for n<5:

Table 2. $ENI(T(K_2 \times P_n))$, for n<5

N	3	4
ENI	6	7

Theorem 3. 5: Let $K_2 \times C_n$ be with 2n vertices torus. Then,

$$ENI(T(K_2 \times C_n)) \geq \lceil \sqrt{6(5n-4)} \rceil - 1.$$

Proof: The number of vertices of the total graph $T(K_2 \times C_n)$ is 5n.In this case function f(m) is $f(m) = m + \dfrac{5n-2m}{\frac{2m-4}{3}}$. Similar to proof of Theorem 3.1, the minimum value of f (m) is $f(m) \geq \sqrt{6(5n-4)} - 1$ and $ENI(T(K_2 \times C_n)) \geq \lceil \sqrt{6(5n-4)} \rceil - 1$.

4. CONCLUSION

Suppose that we want to design a new network with 40 vertices. How do we must design it, according to edge neighbour-integrity and total graph structure?

Table 3. For 40 vertices, some of alternative design.

N=40	The total of the graph	ENI
P_{40}	$T(P_{40}) \rightarrow V(T(P_{40}))=79$	15
C_{40}	$T(C_{40}) \rightarrow V(T(C_{40}))=80$	16
$S_{1,39}$	$T(S_{1,39}) \rightarrow V(T(S_{1,39}))=79$	21
$K_2 \times P_{20}$	$T(K_2 \times P_{20}) \rightarrow V(T(K_2 \times P_{20})=80$	30
$K_2 \times C_{20}$	$T(K_2 \times C_{20}) \rightarrow V(T(K_2 \times C_{20}))=82$	31

By definition of the total graph, the ENI(T(G)) of any graph G which has the most edge is the biggest among the considering structure of the graph with 40 vertices. We must design the new networks as $T(K_2 \times C_{20})$, because its edge neighbour-integrity is greater than the other designs. This construction is more stable all the others.

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