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A NEW APPROACH BY THE HYPERCUBE Q_n ON THE LAST NEW VERTEX VISITED BY A RANDOM WALK

Pınar DÜNDAR

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RASGELE BİR YOLLA ZİYARET EDİLMİŞ EN SON YENİ TEPE ÜZERİNE Q_n HİPERKÜBÜ İLE YENİ BİR YAKLAŞIM

ÖZET

Birleştirilmiş bir G grafının bir tur örtüsü, grafın bir x tepesinden başlar ve her adımda o anda bulunulan tepenin bir komşusuna eşit olasılıkla hareket ederek grafın her tepesini dolaştıktan sonra biter.C_n çevresinin herhangi bir tepesinden başlayan bir tur örtüsünün olasılığı hangi tepede biterse bitsin hep aynıdır.Çalışmalarda , yalnızca C_n çevresi ve K_n tam grafının bu özelliği sağladığı ifade edilmiştir.Bu çalışmada bilgisayar bilimlerinde çok önemli yeri olan Q_n hiperkübünün de aynı özellikte olduğu gösterilmiştir.

SUMMARY

A cover tour of a connected graph G from a vertex x is a random walk that begins at x,moves at each step with equal probability to an neighbour of its current vertex, and ends when it has hit every vertex of G. The cycle C_n is well known to have the curious property that a cover tour from any vertex is equally likely to end at any other vertex ,the complete graph K_n shares this property. In this work we proved that the Q_n hypercube graph has this property, too.

1. INTRODUCTION

A random walk on G , which G is a connected, simple graph, s a stochastic process whose state space consists of vertices of G ,it begins some specified vertex x. At the time 0 the process ;if at time t it is at vertex y, then at time t+1 it will with equal probability be at any z adjacent to y where the probability of any vertex y is the inverse of degree of y(1).

The number of steps required for all vertices to be visited is called cover time of a random walk on G. Computing the expected cover time for G that begins at x vertex is a difficult problem in general, as is determining the distribution of last new vertex to be visited. However, the last new vertex distribution is trivial in the case where G is a cycle C_n A hypercube is a distributed parallel system consisting of 2^n, each provided with its own sizeable memory and interconnected with *n* neighbours. It has a homogeneous symmetric structure and has necessarily rich connectivity. It also has useful topology in which many other topologies, such as meshes, rings(cycles), trees can be embedded(2,3).It is denoted by Q_n

Hypercube structure started to be investigated in the 1970's and provided a good alternative in the operations like Fast Fourier Transform, in similar type Fast Hadamard, Haar, Walsh Transformations, convolution, correlation and matrix operations. The investigations have shown that Fast Fourier Transform algorithmic structure, used as basic algorithm in digital signal processing, in fact proves to be hypercube graph, i.e. they are isomorphic. Hypercube systems were applied to the problems of digital signal processing, image processing, pattern recognition and some other problem solutions and these applications were considered in detail in literature(4,5,6).

The n-dimensional hypercube is a graph consisting 2^n vertices labelled from 0 to

2^n-1.Two vertices(or nodes) are joined by an edge if and only if the binary representations of labels(or addresses),as binary integers, differ one only one bit. Every vertex A has label or address $a_n a_{n-1} ... a_2 a_1$ with $a_i \in \{0,1\}$,where a_i is called the *i*'th bit or *i*'th dimension of the address. Some times *i* is called *i*th co-ordinate. In this case, a_i is called the value of co-ordinate *i*. The important property of hypercube is that, it can be constructed recursively from lower dimensional cubes. Some n-cube graphs are shown figure 1.

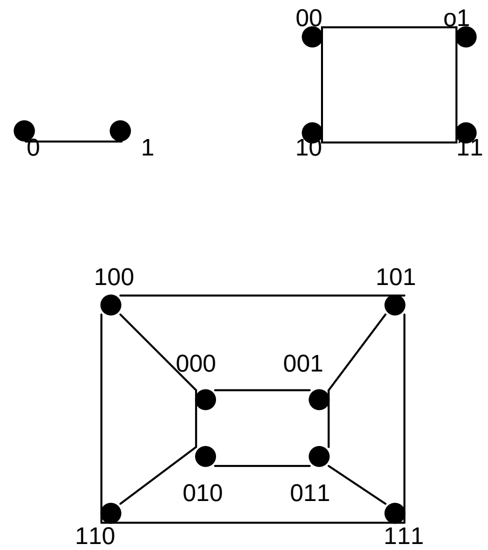

figure1.

1-cube,2-cube and 3-cube.

2. A NEW APPROACH BY THE HYPERCUBE Qn ON THE LAST NEW VERTEX VISITED BY A RANDOM WALK

In this section before is given many definitions and corollary , after are proved some theorems on hypercube graphs which that are used the solution of the problem .

Definition 2.1: In graph theory, the n-dimensional hypercube graph Q_n is defined

$Q_n = K_2 * Q_{n-1}$,where Q_1 is a graph K_2 and K_2 a full connected graph with two vertices , * denotes product of two graph (7).

The n-dimensional a hypercube structure is studied by Dündar(8) ,as the graph structure. The following results are obtained.

Corollary 2.1: Q_n is n-regular graph .

Corollary 2.2: The connectivity of Q_n is n. Thus Q_n is n-connected graph.

Corollary 2.3: The covering number and the independence number of hypercube graph are equal to 2^{n-1}.

Corollary 2.4: The n-dimensional hypercube is a bipartite graph.

Corollary 2.5: If G of order p, k-connected and for some $n \leq k$, every set S of n mutually nonadjacent nodes satisfied $|N(S)| > p*(n-1)/(p+1)$, then G is Hamiltonian(9,10).

Theorem 2.1: The graph Q_n is a Hamiltonian graph.

Proof: The graph Q_n is n-connected graph from Corollary 2.2 and also is

bipartite graph from corollary 2.4.Let V_1 and V_2 are bipartite sets of Q_n . If S set which is talked about in Corollary 2.5 is selected from one of these sets ,it is seen that Corollary 2.5 is obtained. Thus Q_n is a Hamiltonian graph.

Let G connected graph, L(x,y) be the event that y is the last new vertex to be visited by random walk beginning at x vertex where L(x,x)=0.

Corollary 2.6: If G is a cycle ,then for any three distinct vertices x,y and z

Pr(L(x,y))=Pr(L(x,z)) (1).

Corollary 2.7: Let u and v be nonadjacent vertices of a connected graph G. Then there is a neighbour x of u such that $Pr(L(x,v)) \leq Pr(L(u,v))$.

Further, the inequality can be taken to strict if subgraph induced by V(G)-{u,v} is connected (1).

Theorem 2.2: Let G have the property that for any three distinct vertices x , y and z ,

Pr(L(x,y))=Pr(L(x,z)).Then G is isomorphic either to the cycle C_n or the complete graph K_n or the hypercube graph Q_n.

Proof: Suppose G satisfies the condition of theorem. Then G at least must

be 2-connected. Otherwise Pr(L(x,y)) would be 0 for any cut vertex y. Thus we may assume all vertices of G have degree at least 2.

On the other hand, we have from Corollary 2.7, that if u and v are any two nonadjacent vertices of G then G-{u, v} (that is ,the subgraph induced by all vertices of G other than u and v) must be disconnected.

Note that G is not complete (thus its number of vertices is at least 4) it can not have a vertex v of degree p-1, for otherwise the removal of two nonadjacent vertices leaves G connected through v. Thus if G is also not a cycle, it has a vertex of degree between 3 and p-2. Choose a vertex y distinct from and not adjacent to x and let C_1 and C_2 be connected components of G-{x,y}. For i=1,2 choose $z_i \in C_i$ at maximum distance from y in G-{x},then G-{x,z_1,z_2} remains connected. But then G-{z_1,z_2}is connected as well, since at most two neighbours of x have been removed. Since z_1 and z_2 cannot be adjacent, we have a contradiction. The graph Q_n has above property since it is n-connected, n-regular and furthermore Hamiltonian graph(it has a Hamiltonian cycle).

Theorem 2.3: For any three distinct vertices x,y and z of graph Q_n .Then

Pr(L(x,y))=Pr(L(x,z))=1/n.

Proof: Q_n has Hamiltonian path since Q_n is Hamiltonian. Let $t=2^n$ the number of vertices of Q_n and $a=2^n-1$ the number of the edges its any Hamiltonian path. If every step is considered as a output function of stochastic process(11) and

$f_m(m=1,2,...,2^n)$ is probability of branch;

in first step:

$Pr(f_1=r_i|$ where r_i is beginning range$)=n^a/(n^a*t)=1/t$

in second step:

$Pr(f_1|f_2)=Pr(f_2 \text{and } f_1)/Pr(f_1)=(n^{a-1}/(n^a*t))/((n^a/(n^a*t))=1/n$

in third step:

$Pr(f_3|f_2f_1)=Pr(f_3 \text{ and } f_2 \text{ and } f_1)/Pr(f_2 \text{and } f_1)=(n^{a-2}/(n^a*t))/((n^{a-1}/(n^a*t))=1/n$

in a.th step:

$Pr(f_t|f_{t-1}f_{t-2...}f_2f_1)=Pr(f_t \text{ and } f_{t-1} \text{ and } f_{t-2} \text{ and} ...f_2 \text{ and } f_1)/Pr(f_{t-1} \text{ and } f_{t-2} \text{ and } ...f_2 \text{ and } f_1)=(n^0/(n^a*t))/(n^a*t))=1/n$.

Conclusions

In this papers is shown that ,if any problem can be modelling by the hypercube graph, the probability of its cover tour at the everyone step is equal to 1/n in which n denotes degree of hypercube. Since the graph Q_n is n-regular when this path is cut in any step, the probability of path is 1/n, also.

REFERENCES

[1] LOVAZS, L .and WINKLER, P.:A Note On The Last New Vertex Visited By Random Walk, J.Graph Theory 17,no5,41-44,1996.

[2] SAAD,Y. nd SCHULTZ, M. H.: Data Communications Hypercube, J. Parallel Distributed Comput.6,115-135,1989.

[3] SAAD,Y.and SCHULTZ, M.: Topological Properties of Hypercube, IEEE Trans. on Comput.37,867-872,1988.

[4] FOX,G. et all: Solving Problems on Current Processors,Vol.1,General Tech. and Regular Problems,Printece-Hall,1988.

[5] HAYES,J.P. et all: A Microprocessor-based Hypercube Supercomputer, IEEE. Micro,6-16,1986.

[6] PEASE, M.C.: The Direct Binary n-cube Microprocessor Array, IEEE Trans. On Comput.26,5,458-473,1977.

[7] BUCKLEY , F. and HARARY, F.: Distance In Graphs. Addison Wesley Pub. California, 24-25,1990.

[8] DÜNDAR,P.:The Realisation of Boolean Function With Minimum Contact By Graphs. J. of Faculty of Sciences Ege Univ. Serie A, vol 12,2,61-67,1989.

[9] FRAISSE, P.: "A New Sufficient Condition For Hamiltonian Graph", J. Graph Theory 10,405-409,1986.

[10] FAUDRE.,R.J.:"Neigborhood Unions And Hamiltonian Properties Of Graphs", J. Combin. Theory 47B,1-9,1989.

[11] PARZEN, E.: Stochastic Processes**,** Holden Day,SanFrancisco**,**226-239,1962.