ARAŞTIRMA MAKALESİ

ON THE BOUNDARY FORM PERTURBATION METHOD IN THE MECHANICS OF CURVED COMPOSITES

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EĞRİSEL KOMPOSİTLER MEKANİĞİNDE SINIR FORMU PERTURBASYON METODU.

ÖZET:

Eğrisel kompositlerle ilgili problemler sınır formu perturbe metodu ile çeşitli araştırmacılar tarafından incelenmiştir.[1] de bununla ilgili birözetleme mevcuttur.[2] de iki ardışık komposit malzeme için bir problem çözülmüştür.Kompozit tabakalarin üç veya daha fazla olabildiği göz önüne alınarak bu çalışmada bu hal analiz edilmiştir. Yukardaki metodla bulunan sayısal sonuçlar verilmiştir.

SUMMARY

Up to now the boundary form perturbation method for the investigation of problems of curved composites has been developed in the numerous investigations the review of which are given in [1]. The considerable consideration of these problems are made in [2].However in [2] this method has been applied for the case where the material of composite consists of two alternating layers. It is known that, in some cases the material of composite can consist of three or more alternating layers. Taking this situation into account, in the present paper the method [2] is developed for the case where the material of composite consist of the alternating three layers.The numerical results obtained by applying this method are also considered

INTRODUCTION

In many cases for the investigation of the problems of mechanics of curved composites determined as in [2], the boundary form perturbation method is employed. According to this method the contact condition satisfied on the non canonical surfaces are reduced to the conditions satisfied on the coordinate surfaces. In this way the solution of the initial problems are reduced to the solution of the corresponding series problem with contact conditions satisfied on the canonical surfaces. This procedure is described in [2]. However in [2] concrete investigations are made in the case where the material of the composite consists of two alternating layers. In the present paper the method [2] is developed for the case where the material of composite consists of the alternating three layers and applying this method the numerical results related to the stress distribution in the composite are obtained. It is assumed that the concentration of the reinforcing layers is small and considered material is modeled as one curved layers between two varies half space. Moreover it is assumed that the curving of the layer exist only in the direction of Ox_1 axis and the plane strain state is considered.

MATEMATICAL FORMULATION OF THE PROBLEM

We consider an infinite body consisting of two half- space and one curved layer between of these half spaces (figure 1.) The value related to the upper and lower half space, we denote by upper induce(1) and (3) respectively, however the values related to the curved layer-by upper indices (2). We associate Cartesian coordinate system $ox_1x_2x_3$ with the meddle surface of the curved layer and assume that the curving of the layers depend on x_1, only and the thickness of the curved layers is constant (i.e. 2h) in everywhere The equation of the middle surface of this layer we write as fallow

$$x_2 = \varepsilon f(x_1) \qquad (1)$$

where ε is a small parameter and $0 < \varepsilon << 1$. The geometrical meaning of this parameter will be given under describing the form of the function $f(x_1)$ in (1)

We assume that $(\varepsilon df / dx_1) << 1$

Within of the regions(1),(2),(3) (fig.1) we write the full system equations of the theory of elasticity

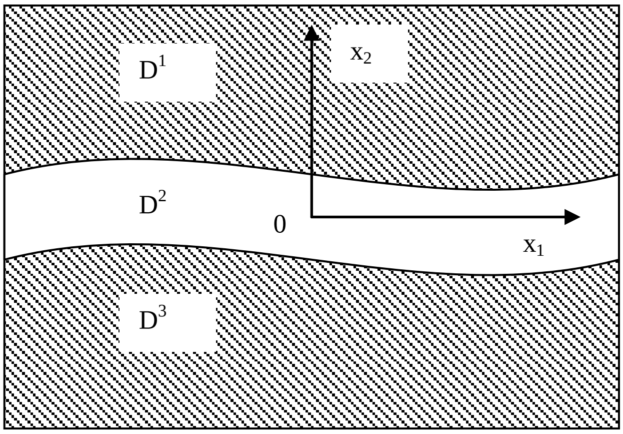

Figure 1.

$$\frac{\partial \sigma_{ij}^{(k)}}{\partial x_j^k} = 0 \qquad \sigma_{ij}^{(k)} = \lambda^{(k)}\theta^{(k)}\delta_{ij} + \mu^{(k)} e_{ij}^{(k)} : \qquad e_{ij}^{(k)} = \tfrac{1}{2}\left(\frac{\partial u_i^{(k)}}{\partial x_j^{(k)}} + \frac{\partial u_j^{(k)}}{\partial x_i^{(k)}} \right);$$

$$\theta^{(k)} = \frac{\partial u_i^{(k)}}{\partial x^k{}_1} + \frac{\partial u_2^{(k)}}{\partial x_2^{(k)}} \qquad (2)$$

in (2) the conventional notation is used. The surfaces contacting the curved layer with the half-space (1) and (3) we denote as S^+ and S^- respectively

Assume that between the layer and half-spaces the following complete contact conditions are satisfied

$$\sigma_{ij}^{(1)}\Big|_{S^+} n_j{}^+ = \sigma_{ij}^{(2)}\Big|_{S^+} n_j{}^+ \qquad \sigma_{ij}^{(2)}\Big|_{S^-} n_j^- = \sigma_{ij}^{(3)}\Big|_{S^-} n_j^-$$

$$u_i^{(1)}\Big|_{S^+} = u_i^{(2)}\Big|_{S^+} \qquad u_i^{(2)}\Big|_{S^-} = u_i^{(3)}\Big|_{S^-} \quad \text{(i,j=1,2)} \qquad (3)$$

In (3) n_j^+, n_j^- are components of ort-normal vector of the surfaces $S^+ and$ S^- respectively.

In the framework of the above-stated we investigate the stress-strain distribution in the body for the case where this body is loaded at $|x_1| \to \infty$ by the uniformly distributed normal forces with the intensity $\sigma_{11} = p$ which acts in the direction of the Ox_1 axis. Thus with the above stated the formulation of the problem is exhausted

METHOD Of SOLUTION

First of all using (1) and constantansy condition of the thickness of the curved layer we derive the following equation of the surfaces S^+, S^- :

$$x_1^{\pm} = t \mp \frac{h}{L(t)}\frac{dF}{dt} \quad x_2^{\pm} = F\ (t) \pm \frac{h}{L(t)} \ L(t) = \left[1 + \left(\frac{dF(t)}{dt}\right)^2\right]^{\frac{1}{2}} \qquad (4)$$

$$F(t) = \varepsilon f\,(t)$$

where t is a parameter $t \in (-\infty, +\infty)$ and h is the half –thickness of the curved layer. According to (4) the following expression are obtained for $n^{\pm}$,

$$n_1^{\pm} = \frac{-\dfrac{dx_2}{dt}}{A^{\pm}(t)}; \qquad n_2^{\pm} = \frac{\dfrac{dx_1}{dt}}{A^{\pm}(t)}$$

where

$$A^{\pm}(t)=\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2\right]^{\frac{1}{2}} \quad (5)$$

Taking the condition $|\varepsilon df/dx_1|<<1$ into account the equation (4) and (5) can be expressed as follows

$$x_1^{\pm}=t\pm h\sum_{k=1}^{\infty}\varepsilon^k a_k(t) \qquad x_2^{\pm}=\pm h+\sum_{k=1}^{\infty}\varepsilon^k b_k(t) \quad (6)$$

$$n_1^{\pm}=\pm h\sum_{k=1}^{\infty}\varepsilon^k d_k(t) \qquad n_2^{\pm}=1\pm h\sum_{k=1}^{\infty}\varepsilon^k g_k(t)$$

According to (6) we represent the sought values in series form in ε

$$\sigma_{ij}^{(k)}=\sum_{q=0}^{\infty}\varepsilon^q\sigma_{ij}^{(k),q} \qquad e_{ij}^{(k)}=\sum_{q=0}^{\infty}\varepsilon e_{ij}^{(k),q} \qquad u_i^{(k)}=\sum_{q=0}^{\infty}\varepsilon^q u_i^{(k),q} \quad (7)$$

Due to linearity the equations (2) are satisfied for each approximations (7) separately.

Substituting (7) in (3) and taking (6) into account and doing some operation, we obtain the following equations from the contact condition (3)

$$\sum_{n=0}^{\infty}\varepsilon^n\left[\sigma_{i2}^{(1),n}-\sigma_{i2}^{(2),n}+G_{In}\left(\sigma_{i1}^{(1),n-1},...\sigma_{i1}^{(1),0},\sigma_{i2}^{(1),0}\sigma_{i2}^{(2),0},\tfrac{\partial f}{\partial t},\frac{\partial^n f}{\partial t^n}\right)\right]=0$$

$$\sum_{n=0}^{\infty}\varepsilon^n\left[u_i^{(1),n}-u_i^{(2),n}+Q_{in}\left(u_i^{(1),n-1},...,u_i^{(2),0},u_i^{1,0};\tfrac{\partial f}{\partial t},...,\frac{\partial^n f}{\partial t^n}\right)\right]\Big|\;(t,h)\Big|=0$$

$$\sum_{n=0}^{\infty}\varepsilon^n\left[\sigma_{i2}^{(1),n}-\sigma_{i2}^{(2),n}+G_{In}\left(\sigma_{i1}^{(1),n-1},...\sigma_{i1}^{(1),0},\sigma_{i2}^{(1),0}\sigma_{i2}^{(2),0},\tfrac{\partial f}{\partial t},\frac{\partial^n f}{\partial t^n}\right)\right]=0 \quad (8)$$

$$\sum_{n=0}^{\infty}\varepsilon^n\left[u_i^{(1),n}-u_i^{(2),n}+Q_{in}\left(u_i^{(1),n-1},...,u_i^{(2),0},u_i^{1,0};\tfrac{\partial f}{\partial t},...,\frac{\partial^n f}{\partial t^n}\right)\right]\Big|\;(t,h)\Big|=0$$

The expression of the functions G_{in},Q_{in} are given under consideration for concrete approximation. Thus we can obtain from (8) the contact condition for each approximation (7). In this case we observe that the contact condition written for the q-th approximation contain the values of all previous approximations. We write the contact condition for the zeroth, first ,and second approximations the zeroth approximation;

$$\sigma_{i2}^{(1),0}\Big|_{(t,h)}=\sigma_{i2}^{(2),0}\Big|_{(t,h)} \qquad \sigma_{i2}^{(2),0}\Big|_{(t,-h)}=\sigma_{i2}^{(3),0}\Big|_{(t,-h)}$$

$$u_i^{(1),0}\Big|_{(t,h)}=u_i^{(2),0}\Big|_{(t,h)} \qquad u_i^{(2),0}\Big|_{(t,-h)}=u_i^{(3),0}\Big|_{(t-h)} \quad (9)$$

the first approximation:

$$\left(\sigma_{i2}^{(1),1}-\sigma_{i2}^{(2),1}\right)\Big|_{(t,h)}=(\sigma_{i1}^{(2),0}-\sigma_{i1}^{(1),0})\Big|_{(t,h)}\cdot\frac{\partial f}{\partial t}$$

$$(\sigma_{i2}^{(2),1}-\sigma_{i2}^{(3),1})\Big|_{(t,-h)}=(\sigma_{ii1}^{(2),0}-\sigma_{i2}^{(3),0})\Big|_{(t,-h)}\frac{df}{dt} \qquad (10)$$

$$(u_i^{(1),1}-u_i^{(2),1}\big|(t,+h)=\left(\frac{\partial u_i^{(2),0})}{\partial t}-\frac{\partial u_i^{(1),0}}{\partial t}\right)\Big|(t,+h).\frac{\partial f(t)}{\partial t}+\left(\frac{\partial u_i^{(2),0}}{\partial x}-\frac{\partial u_i^{(1),0}}{\partial x}\right)\Big|(t,+h)$$

the second approximation:

$$\left(\sigma_{12}^{(1),2}-\sigma_{12}^{(2),2}\right)\Big|_{(t,h)}=2\pi\sin(2\alpha x_1)\left[\psi_{11}^{(1),1}-\psi_{11}^{(2),1}\right]+1/2\alpha h\left(\psi_{12}^{(2),1}-\psi_{12}^{(1),1}\right)$$

$$\left(\sigma_{12}^{(2),2}-\sigma_{12}^{(3),2}\right)\Big|_{(t,h)}=2\pi\sin(2\alpha x_1\left[\psi_{11}^{(2),1}-\psi_{11}^{(3),1}\right]+1/2\alpha h\left(\psi_{12}^{(3),1}-\psi_{12}^{2),2}\right)$$

$$\left(\sigma_{22}^{(1),2}-\sigma_{22}^{(2),2}\right)\Big|_{(t,h)}=2\pi\cos(2\alpha x_1)\left[\psi_{12}^{(1),1}-\psi_{11}^{(2),1}\right]$$

$$\left(\sigma_{22}^{(2),2}-\sigma_{22}^{(3),2}\right)\Big|_{(t,h)}=2\pi\cos(2\alpha x_1)\left[\psi_{11}^{(2),1}-\psi_{11}^{(3),1}\right]$$

$$(u_1^{(1),2}-u_1^{(2),2}\big|(t,+h)=\tfrac{l}{2}\sin(2\alpha x_1)\left(\frac{\partial\varphi_i^{(2),1})}{\partial x_2^{(2)}}-\frac{\partial\varphi_1^{(2),1}}{\partial x_2^{(1)}}\right)\Big| \;.$$

$$(u_1^{(2),2}-u_1^{(3),2}\big|(t,+h)=\tfrac{l}{2}\sin(2\alpha x_1)\left(\frac{\partial\varphi_i^{(2),1})}{\partial x_2^{(2)}}-\frac{\partial\varphi_1^{(3),1}}{\partial x_2^{(1)}}\right)\Big| \;. \qquad (11)$$

$$(u_2^{(1),2}-u_2^{(2),2}\Big|_{|(t,\pm h)}=\pi\cos(2\alpha x_1)\left(\pm\alpha\;\; h'^{(2)}\left(\varphi_2^{(2),1}-\varphi_2^{(1),1}\right)-\right)*\tfrac{l}{2}\left(\tfrac{\partial\varphi_2^{21}}{dx_2^2}-\tfrac{\partial\varphi_2^{11}}{dx_2^1}\right)$$

$$\pm\alpha*h(\varphi_2^{21}-\varphi_2^{11})+\tfrac{l}{2}\left(\tfrac{d\varphi_2^{21}}{dx_2^2}-\tfrac{d\varphi_2^{11}}{dx_2^1}\right)\Big|_{(\pm h)}$$

$$(u_2^{(2),2}-u_2^{(3),2}\Big|_{|(t,\pm h)}=\pi\cos(2\alpha x_1)\left(\pm\alpha\;\; h'^{(2)}\left(\varphi_2^{(2),1}-\varphi_2^{(1),1}\right)-\right)*\tfrac{l}{2}\left(\tfrac{\partial\varphi_2^{21}}{dx_2^2}-\tfrac{\partial\varphi_2^{11}}{dx_2^1}\right)$$

$$\pm\alpha*h(\varphi_2^{21}-\varphi_2^{11})+\tfrac{l}{2}\left(\tfrac{d\varphi_2^{21}}{dx_2^2}-\tfrac{d\varphi_2^{11}}{dx_2^1}\right)\Big|_{(\pm h)}$$

We can also write the third, fourth and subsequent approximation in the same manner.
Now we consider the determination procedure of the mentioned approximations.
After some operations the values of the zeroth approximation are determined from (2),(9) as follows.

$$u_1^{(k),0}=\frac{\sigma_{11}^{(k),0}(1-\nu^k)}{E_k}x_1 \quad u_2^{(k),0}=-\frac{\nu_k(1+\nu_k)}{E_k}\sigma_{11}^{(k),0}x_2+c^{(k)}$$ (k=1,2,3)

$$\sigma_{11}^{(1),0}=p\left[\tfrac{1}{2}(\left(1+\frac{(1-\nu_1)E_3}{(1-\nu_3)E_1}\right)\right]^{-1}$$

$$\sigma_{11}^{(2),0}=p\frac{E_2(1-\nu_1)}{E_1(1-\nu)}\left[\tfrac{1}{2}\left(1+\frac{(1-\nu_1)E_3}{(1-\nu_3)E_1}\right)\right]^{-1}$$

$$\sigma_{11}^{(3),0}=p\frac{(1-\nu_1)E_3}{(1-\nu_3)E_2}\left[\tfrac{1}{2}\left(1+\frac{E_3(1-\iota)}{E_2(1-\nu_3)}\right)\right]^{-1} \quad (12)$$

For determination of the first and second approximation we use Papkovıch-Neiber representations which are written in the following form for the displacement

$$2\mu^{(k)}u_i^{(k),q}=-\frac{\partial\Phi_0^{(k),q}}{\partial x_1}-x_2^{(k)}\frac{\partial\Phi_2^{(k),q}}{\partial x_2^{(k)}}$$

$$2\mu^{(k)}u_2^{(k),q}=(3-4\upsilon^{k)})\Phi_2^{(k),q}-\frac{\partial\Phi_0^{(k),q}}{\partial x_2^{(k)}}-x_2^{(k)}\frac{\partial\Phi_2^{(k),q}}{\partial x_2^{(k)}} \quad (13)$$

In (13) $\upsilon^{(k)}$ Poisson's ratio , $\mu^{(k)}$ is Sheare modulus. $\Phi_0^{(k),q}$ and $\Phi_2^{(k),q}$ is a Harmonic function. In the present paper we assume that the curving of the reinforcing layer (fıg.1.) is a periodic one and the function (4) is select as follows .

$$F(x_1)=L\sin\tfrac{2\pi}{l}x_1=\varepsilon l\sin\tfrac{2\pi}{l}x_1 \quad (14)$$

ıt is assumed that L<< l and introduced a small parameter ε=L/l. According to (10), (14) and (13) the harmonic function $\Phi_0^{(k),q}$ and $\Phi_2^{(k),q}$ is selected as follows

$$\Phi_0^{(k),q}(x_1,x_2)=\varphi^{(k),q}{}_0(x_2)\sin\alpha x_1\,;\Phi_2^{(k),q}(x_1,x_2)=\varphi_2^{(k),q}(x_2)\sin\alpha x_1 \quad (15)$$

by the ordinary procedure the function $\varphi_0^{(k),q},\varphi_2^{(k),q}$ are determined in the following form,

$$\varphi_q^{(1),1}(x_2)=A_{1q}e^{-\alpha x_2} \qquad \varphi_q^{(2),1}(x_2)=A_{2q}ch\alpha x_2+A_{3q}sh\alpha x_2$$

$$\varphi_q^{(3),1}(x_2)=A_{4q}e^{\alpha x_2} \quad q=0,2 \quad (16)$$

Using (15), (16) we determine the expressions for the stress and displacement of the first approximation and for the determination of the unknown constant in(16) the linear algebraic equation from the contact condition (10) are obtained in this way we determine the values of the first approximation . Continuing this procedure in the similar manner we determine the value of the second and subsequent approximations.

NUMERICAL RESULTS

Now we consider some numerical results which are obtained in the framework of the foregoing methods .Note that these result relate to the distributions of the normal and tangential stresses on the interface of the layer and half planes. We assume that the material of the half plane-1 differs from the material of the layer-2 and of the half plane-3 (fıg.1.) Moreover we assume that $\upsilon^{(1)} = \upsilon^{(2)} = \upsilon^{(3)} = 0.3$. Where $\upsilon^{(k)}$ is a Poissons ratio .The modulus of these component we denote as $E^{(k)}$ In fıg.2,3 the graph of the relation between $\frac{\sigma_{nn}}{p}$ and αx_1 are given for various $\frac{E^2}{E^1}, \frac{E^2}{E^3}$

Note that in fıg.2 σ_{nn} mean the normal stress on the S^+ surface (fıg.1),

But in fig.3 σ_{nn} means the normal stress on the S^- surface(fıg.1) The distribution of the share stress σ_{nt} on the S^- surface is given in fıg.4

In these figs. the graphs obtained in the case for which $\frac{E^{(2)}}{E^{(1)}} = \frac{E^{(2)}}{E^{(3)}}$ coincide with the corresponding ones given in [2]. It follows from these results that for fixed $\frac{E^{(2)}}{E^{(1)}}$ the values of σ_{nn} / p ,and σ_{nt} / p increase monotonically with $\frac{E^{(2)}}{E^{(3)}}$ .Moreover the character of the obtain distributions agree with corresponding ones given in[2]. In table 1. the values of σ_{nn} / p are given for various approximations in this table 1

σ_{nn}, σ_{nt} are calculated at $\alpha x_1 = \frac{\pi}{2}$, $\alpha x_1 = 0$ respectively with various $\frac{E^{(2)}}{E^{(1)}}, \frac{E^{(2)}}{E^{(3)}}$ .It follows from this table that the convergent of the numerical results has the high accuracy

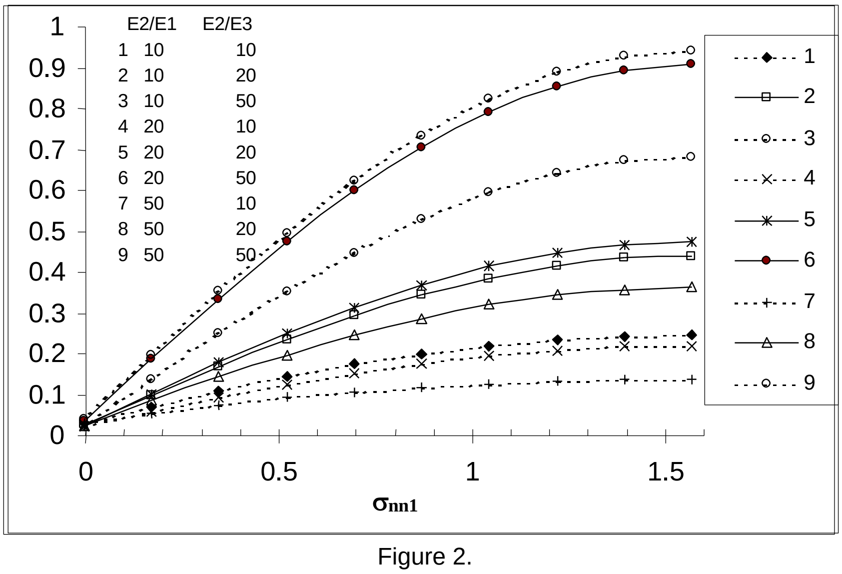

Figure 2.

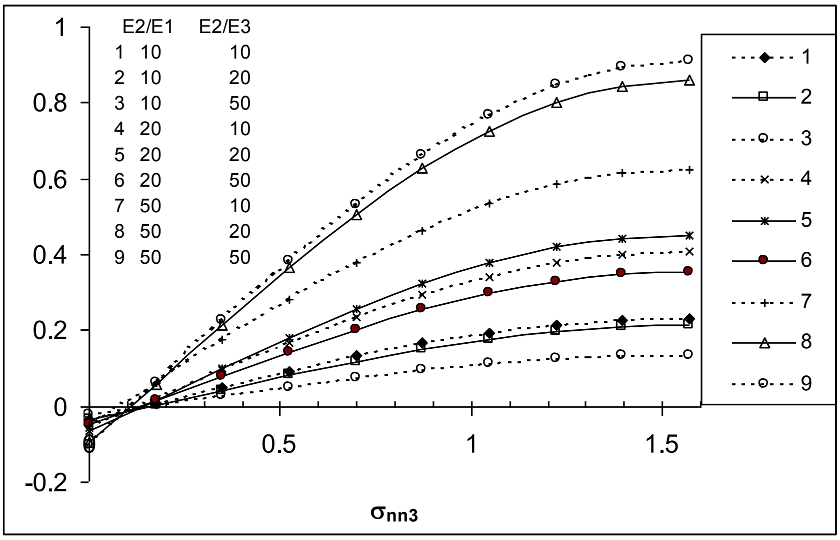

Figure 3.

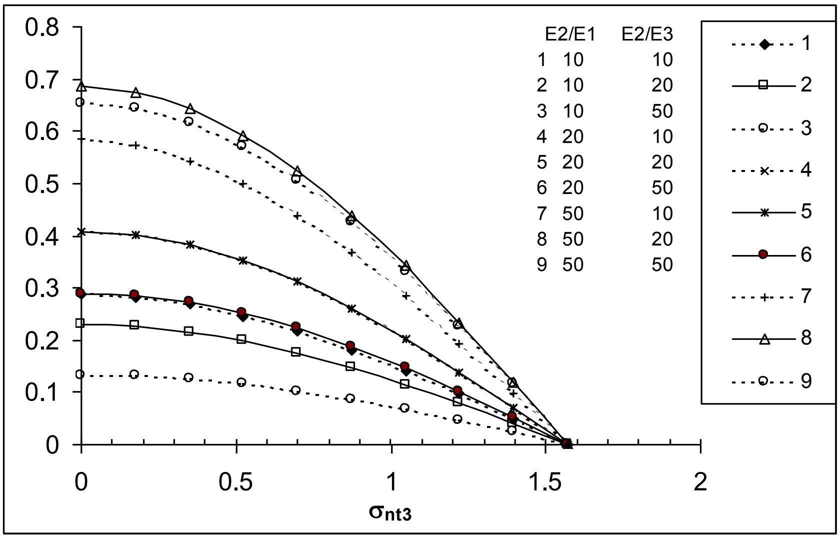

Figure 4.

Table 1.

E1/E2	E2/E3	σ_{nn} / p		σ_{nt} / p	
10	10	1 2	0.2385 0.2440	1 2	0.2870 0.2870
10	20	1 2	0.4216 0.4361	1 2	0.4248 0.4248
10	50	1 2	0.6526 0.6802	1 2	0.5841 0.5841
50	10	1 2	0.1345 0.1340	1 2	0.1319 0.1319
50	20	1 2	0.3566 0.3591	1 2	0.2874 0.2874
50	50	1 2	0.9241 0.9377	1 2	0.6517 0.6517

REFERENCES:

[1] Akborov,S.D.,Güz,A.N.and Yahnioğlu,N.Mechanics of compozite metarials with curved structures and elements of consructions (review).İntern.Appl.Mech.Vol.34,no 11,pp.1067-1079.

[2] AKBAROV,S.D.and GÜZ,A.N. Mechanics of curved composites.Kluwer Academic Puplishers,2000,464p.