ARAŞTIRMA MAKALESİ

A STUDY RELATED TO STOCHASTIC MODEL DETERMINATION IN GEODETIC NETS

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JEODEZİK AĞLARDA STOKASTİK MODEL BELİRLENMESİNE İLİŞKİN BİR ÇALIŞMA

ÖZET

Klasik jeodezide nokta koordinatları için nirengi ağları, nokta yükseklikleri için nivelman ağları (nirengi ağlarının hesabı için gereken nokta yükseklikleri)oluşturulur. Nivelman ve nirengi ağları olarak adlandırılan jeodezik ağlarda farklı alet ve ölçme yöntemleri kullanılır. Ayrıca, yeni ölçme teknolojileri ve klasik ölçmeler birlikte de kullanılmaktadır. Bu nedenle farklı alet ve ölçme yöntemleri ile ölçülen jeodezik ağlarda bir stokastik model tanımlamak oldukça zordur. Bu çalışmada "Helmert Varyans Bileşen Tahmin Yöntemi" tanıtılmış, nivelman ve nirengi ağları için dengeleme sonuçları verilmiştir.

SUMMARY

In conventional geodesy , triangulation nets for coordinates of points and leveling nets for heights of points are formed (heights are needed for calculation of the triangulation nets). Different instruments and measurement methods are used in geodetic networks which are named leveling and triangulation nets. The new measurement technologies are also used with conventional measurements. It is known that it is too difficult to define a stochastic model in geodetic networks which are measured by different instruments and measurement methods. In this study, "Variance Estimation by Helmert Type" is introduced and the adjustment results are given for both leveling net and triangulation net.

1. INTRODUCTION

Precise leveling is the main technique in measurements of leveling nets. However, because of the topography of the land (i.e. river, valley crossing) some height differences in leveling nets can not be measured by precise leveling. As known, these height differences can also be measured by other methods (trigonometric or valley cross leveling). Thus, measurements of complex leveling nets measured by different measurements and methods are obtained. Measurements of conventional triangulation nets are similar to complex leveling nets and these triangulation nets are formed which are measured by lengths and directions. In addition, GPS (Global Positioning System) measurements are also used along with conventional triangulation nets. So, stochastic models which take into consideration the methods of measurements and important sources of errors affecting accuracy, should be formed, in geodetic nets.

In this study, "variance component estimation of Helmert type" is introduced and the results of both triangulation and leveling nets are given.

2. FUNCTIONAL AND STOCHASTIC MODELS OF ADJUSTMENT

Adjustment of geodetic nets is performed in two ways called as conditional and indirect approaches. In practice, the indirect approach is the most commonly used method. The basic mathematical model is

$$E(\underline{l}) = \underline{A}\underline{x} \tag{1}$$

and the stochastic model is

$$C(\underline{l}) = \sigma_0^2 \underline{P}^{-1} \tag{2}$$

where $E(\underline{l})$ is expected value of observation, $\underline{A}$ is the $n \times u$ coefficient matrix, $\underline{x}$ is the $u \times 1$ vector of unknown parameters, $C(\underline{l})$ is the $n \times n$ variance-covariance matrix of observations, σ_0^2 is variance of observation (in unit weight), $\underline{P}^{-1}$ is weight matrix (Grafarend and Schaffrin 1974; Wolf 1975; Torge 1980; Kok 1983; Niemmeier 1983; Niemmeier 1985; Ebong 1987;).

2.1 Conventional Stochastic Model For Leveling And Triangulation Nets

As well-known stochastic model for both leveling and triangulation nets is

$$P_i = \frac{C}{\sigma_i^2} \tag{3}$$

The weight given by (3), can be used in adjustment of leveling nets and triangulation nets (Yavuz 2000; Coşkun 1996; Grafarent and Schaffrin 1974; Öztan 1982; Ulsoy 1963; Vanicek and Grafarent 1980). A-priori variance of observation can be derived from repeated measurements. The constant, C can be taken as 1. There are several stochastic models in leveling or complex leveling nets (Coşkun 1996). Only one of them (i.e: 3) is used as a conventional stochastic model in examining of leveling nets because the stochastic model (equation 3) can be used for both adjustment of leveling and triangulation nets (Yavuz 2000).

3. VARIANCE COMPONENT ESTIMATION OF HELMERT TYPE

The variance component estimation of Helmert type is summarized in steps below.

Step 1*.* Observations are divided into a number of groups of m according to the measurement methods and the weight matrix of every group is estimated before adjustment. The weight can be chosen as identity matrix (Sahin 1993; Yavuz 2000),

$$\underline{P}_1 = \underline{P}_2 = ... = \underline{P}_m = \underline{I} \quad \text{(identity matrix)} \tag{4}$$

Step 2. Global Matrix $\underline{N}$ and normal matrix $\underline{N}_1, \underline{N}_2, ..., \underline{N}_m$ of every group are formed by using the estimated initial weights.

$$\underline{N}_k = \underline{A}_k^T \underline{P}_k \underline{A}_k \quad (k = 1, 2, ..., m) \tag{5}$$

$$\underline{N} = \underline{A}^T \underline{P} \underline{A} = \underline{N}_1 + \underline{N}_2 + ... + \underline{N}_m \tag{6}$$

Where, $\underline{A}_k$ is the coefficient matrix of the group of k, $\underline{A}$ is the coefficient matrix of all observations.

Step 3. From the least square solution, the unknown parameters and the observation residuals are computed as follows

$$\underline{\hat{x}} = \underline{N}^{-1} A^T \underline{P} \underline{l} \tag{7a}$$

$$\underline{v}_k = \underline{A}_k \underline{\hat{x}} - \underline{l}_k \quad (k = 1, 2, ..., m) \tag{7b}$$

Where

$\underline{\hat{x}}$	: estimation value of *unknown parameters*
l	: *observations vector of all groups*
$\underline{l}_k$	: *observations vector of group of* k
$\underline{v}_k$	: *residuals vector of group of* k

Step 4. Then the following Helmert equation is formed as

$$\begin{bmatrix} h_{11} & h_{12} & ... & h_{1m} \\ h_{21} & h_{22} & ... & h_{2m} \\ . & . & ... & . \\ h_{m1} & h_{m2} & ... & h_{mm} \end{bmatrix} \begin{bmatrix} \overline{\sigma}_1^2 \\ \overline{\sigma}_2^2 \\ . \\ \overline{\sigma}_m^2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ . \\ c_m \end{bmatrix} \tag{8}$$

Where,

$$c_k = \underline{v}_k^T \underline{P}_k \underline{v}_k \tag{9a}$$

$$h_{kk} = n_k - 2Tr(\underline{N}^{-1} \underline{N}_k) + Tr(\underline{N}^{-1} \underline{N}_k)^2 \tag{9b}$$

$$h_{kj} = Tr(\underline{N}^{-1} \underline{N}_j \underline{N}^{-1} \underline{N}_k) \quad \text{(for } k \neq j\text{)} \tag{9c}$$

where $\overline{\sigma}_k^2$ $(k = 1, 2, ..., m)$ variance of group of k.

Step 5. Having solved for $\overline{\sigma}_1^2, \overline{\sigma}_2^2, ..., \overline{\sigma}_m^2$, then the solution is performed by the iteration since the values of c_k are dependent of P_k which is a function of variance $\overline{\sigma}_k^2$. If $\overline{\sigma}_j^2$ is not equal to 1 for all A_i — B_i, the procedure returns to step 2. When $\overline{\sigma}_j^2 = 1$ for all m groups the iteration stops (Welsh 1978; Grafarent et al 1980; Şahin 1993; Yavuz 2000).

4. CRITERIA FOR EXAMINING THE STOCHASTIC MODELS

In order to examine the stochastic models, criteria have to be defined. Stochastic model, homogeneity and mean accuracy tests can be used for leveling and triangulation nets. In addition, isotropy test can be used for triangulation nets (Yavuz 2000). The mathematical principles of these tests are explained as follows for the two nets.

4.1 Stochastic Model Test

Variance (a-priori) of σ_0^2 can be determined by either pre-adjustment or experiences. Variance (a-posteriori) of $\overline{\sigma}_0^2$ determined after adjustment, must be in agreement with the variance of σ_0^2 determined before adjustment. In this case, the hypothesis is

$$H_0 = E(\sigma_0^2) = E(\overline{\sigma}_0^2) \tag{10}$$

The test magnitude is the ratio of greater variance to smaller one.

$$T = \frac{\overline{\sigma}_0^2\,(f_1)}{\sigma_0^2\,(f_2)} \quad \text{or} \quad T = \frac{\sigma_0^2\,(f_2)}{\overline{\sigma}_0^2\,(f_1)} \tag{11}$$

$F_{1-\alpha, f_1, f_2}$ is derived from the F_Table for the degrees of freedom $f_1 = n - u + d$, $f_2 = \infty$ and the confidence limit ($1-\alpha$).

$$T > F_{1-\alpha, f_1, f_2} \tag{12}$$

zero hypotheses is rejected, in other words, weights of observations are not chosen properly (Wolf 1975).

4.2 Homogeneity Test For Leveling Nets

The homogeneity criterion is the standard deviation of the standard deviation of the heights. Since the standard deviation (σ_J, $J = 1,...,u$) of a number of u heights are calculated by adjustment with the use of any stochastic model, the standard deviation of the mean standard deviation can be calculated by,

$$\mu = \sqrt{\frac{\sum_{j=1}^{u}(\sigma^0 - \sigma_j)^2}{u(u-1)}} \qquad (13)$$

where

$$\sigma^0 = \frac{\sum_{j=1}^{u}\sigma_j}{u} \qquad (14)$$

After the values of μ and σ^o are computed for a number of k stochastic models, the homogeneity test is applied by comparing the values of μ for every two pairs. The values of μ_i and μ_j related to the two stochastic models, the zero hypothesis is formed as

$$H_0 = E(\mu_i^2 - \mu_j^2) = 0 \quad , \quad i, j = 1,...,k \ (i \neq j) \qquad (15)$$

The test value of

$$T_{i,j} = \frac{\mu_i^2}{\mu_j^2} \ , \ \mu_i^2 > \mu_j^2 \qquad (16)$$

is compared with the F value derived from the F-Table for the degrees of freedom ($f_i = f_j = u-1$) and the confidence limit ($1-\alpha$). When

$$T_{ij} \leq F_{1-\alpha, f_i, f_j} \qquad (17)$$

the zero hypotheses is valid. When

$$T_{i,j} = \frac{\mu_i^2}{\mu_j^2} > F_{1-\alpha, f_i, f_j} \quad , \quad \mu_i^2 > \mu_j^2 \qquad (18)$$

the stochastic model having μ_j is superior than the other, based on homogeneity (Coşkun 1996; Yavuz 2000).

4.3 Mean Accuracy Test for Leveling Nets

In order to classify the stochastic models whose superiorities can not be defined (i.e. homogeneity zero hypothesis), the mean accuracy criterion is used. In such case, the zero hypothesis is

$$H_0 = (\sigma_i^0) = (\sigma_j^0) \tag{19}$$

and the test value is

$$t_{ij} = \frac{\sigma_i^0 - \sigma_j^0}{\sqrt{\mu_i^2 + \mu_j^2}} \tag{20}$$

for the degrees of freedom $f = 2(u-1)$ and the confidence limit ($1-\alpha$). If the value of t_{ij} is compared with the table value ($t_{1-\alpha,f}$) and if

$$t_{ij} \leq t_{1-\alpha,f} \tag{21}$$

the zero hypothesis is valid. So the two stochastic models have the equal priorities. If

$$t_{ij} = \frac{\sigma_i^0 - \sigma_j^0}{\sqrt{\mu_i^2 + \mu_j^2}} > t_{1-\alpha,f} \quad , \quad (\sigma_i^0 > \sigma_j^0) \tag{22}$$

the model which has smaller mean standard deviation is more appropriate (Coşkun 1996; Yavuz 2000).

4.4 Homogeneity Test For Triangulation Nets

The standard deviation calculated from the sum of short axis and long axis of error ellipse can be used as homogeneity criterion (Yavuz 2000). Since the axis of error ellipse (A_i and B_i , $i = 1,2,...,u$) of a number of u position of point are calculated by adjustment with the use of any stochastic model, then the standard deviation of the summation can be calculated by

$$\tau = \sqrt{\frac{\sum_{i=1}^{u} (\Delta_0 - \Delta_i)^2}{u(u-1)}} \tag{23}$$

where

$$\Delta_0 = \frac{\sum_{i=1}^{u} \Delta_i}{u} \quad , \quad \Delta_i = A_i + B_i \tag{24}$$

After the values of τ is computed for a number of k stochastic models, the isotropy test is applied by comparing the values of τ for every two pairs. The values of τ_i and

τ_j related to the two stochastic models, the zero hypothesis is formed as

$$H_0 = E(\tau_i^2 - \tau_j^2) = 0 \quad , \quad i, j = 1, 2, ..., m \; (i \neq j) \tag{25}$$

The test value of

$$T_{i,j}^{\Delta} = \frac{\tau_i^2}{\tau_j^2} \quad (\tau_i^2 > \tau_j^2) \tag{26}$$

is compared with the F value derived from the F-Table for the degree of freedom ($f_i = f_j = u - 1$) and the confidence limit ($1-\alpha$). When

$$T_{i,j}^{\Delta} \leq F_{1-\alpha, f_i, f_j} \tag{27}$$

the zero hypothesis is valid. When

$$T_{i,j}^{\Delta} > F_{1-\alpha, f_i, f_j} \tag{28}$$

the stochastic model having τ_j is superior than the other, based on homogeneity.

4.5 Isotropy Test for Triangulation Nets

The standard deviation of differences between short and long axis of error ellipses (A_i , B_i) can be used as isotropy test (Yavuz 2000). The standard deviation is

$$\eta = \sqrt{\frac{\sum_{i=1}^{u} (\omega_0 - \omega_i)^2}{u(u-1)}} \tag{29}$$

where

$$\omega_0 = \frac{\sum_{i=1}^{u} \omega_i}{u} \quad , \quad \omega_i = A_i - B_i \tag{30}$$

After values of η is computed for a number of m stochastic models, the isotropy test is applied by the values of η for every two pairs. The values of η_i and η_j related to the two stochastic models, the zero hypothesis is formed as

$$H_0 = E(\eta_i^2 - \eta_j^2) = 0 \quad , \tag{31}$$

The test value

$$T_{i,j}^{\eta} = \frac{\eta_i^2}{\eta_j^2} \quad (\eta_i^2 > \eta_j^2) \tag{32}$$

is compared with the F value derived from the F-Table for the degree of freedom $f_i = f_j = u-1$ and the confidence limit ($1-\alpha$). When

$$T_{i,j}^{\eta} \le F_{1-\alpha, f_1, f_2} \tag{33}$$

the zero hypothesis is valid. When

$$T_{i,j}^{\eta} = \frac{\eta_i^2}{\eta_j^2} > F_{1-\alpha, f_1, f_2} \quad (T = \sigma_0^2 / \hat{\sigma}_0^2) \tag{34}$$

the stochastic model having η_j is superior than the other, based on isotropy.

The mean accuracy tests can be applied as in the mean accuracy test for the levelling nets explained in section 4.3 (equations from 19 to 22) for the homogeneity of triangulation and the isotropy of triangulation nets.

5. EXPERIMENTS

A leveling net and two triangulation nets are used as example nets for adjustment using both the conventional stochastic model and the variance component estimation of Helmert type. The results of adjustment are given for both stochastic models. The adjustment for the first stochastic model (eq3), is named (I). The adjustment for the second stochastic model, which is the variance component estimation of Helmert type, is named as II.

The criteria for leveling nets in our stochastic model are;

1- Stochastic model test
2- Homogeneity test
3- Mean accuracy test (if the results of the homogeneity test for both stochastic models are similar)

The Criteria for triangulation nets used, are;

1- Stochastic model test
2- Homogeneity test
3- Mean accuracy test (if the results of the of the homogeneity test for both stochastic models are similar)
4- Isotropy test
5- Mean accuracy test (if the results of the isotropy test (criteria 4) for both stochastic models are similar)

5.1 Experiments of Leveling Net

The data which were collected for a PhD study in the University of Selçuk, in Konya have the following characteristics. The net has 8 benchmarks and 15 leveling lines ranging from 120 to 650 m (Figure 1).

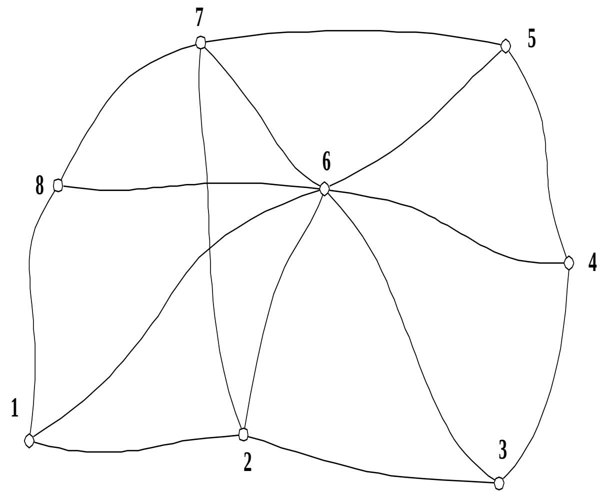

Figure 1: Konya Leveling Net

The net was measured by both precise and trigonometric leveling methods. Reciprocal trigonometric leveling and leap-frong trigonometric leveling methods are used in measurements of trigonometric leveling. The three different measurement methods (precise leveling, leap-frong trigonometric leveling and reciprocal trigonometric leveling) are used for every leveling line. Leveling data are adjusted by least-square adjustment with free adjustment approach using both stochastic models (Model I and Model II). A-priori and a-posterior variances are shown in Table 1.

Table 1: Stochastic Model Test ($F_{0.95,38,\infty} = 1.404$)

Stochastic Models	Before Adjustment	After Adjustment	Test Value	Zero Hypothesis
I	1.000	1.883	3.546	Not Accepted
II	1.000	1.000	1.000	Accepted

The standard deviations for heights, homogeneity and mean accuracy tests (for both stochastic models) are listed in Table 2.

Table 2: Results of Konya Leveling Net

	Stochastic Models		
	I (mm)	II (mm)	Explanations
1	0.458	0.152	Standard deviations of heights after Adjustment (m_{H_i})
2	0.369	0.123	
3	0.401	0.134	
4	0.464	0.154	
5	0.443	0.147	
6	0.308	0.100	
7	0.432	0.144	
8	0.466	0.155	
σ^0	0.417	0.139	Mean standard deviation of standard deviations
μ	0.020	0.007	Standard deviation of standard deviations
	$T_{I,II} = 8.163$		The test values of the homogeneity test

The results tabulated in Table 2 indicates that model I does not accept the zero hypothesis and it is not a convenient model. Model II however, is a convenient model and it accepts zero hypothesis (X>1).

The second criterion (homogeneity test), which is applied to the results of leveling net adjustment is calculated for both the stochastic models. The test value is $T_{I,II} = 8.163$ which is computed from the equation 15 and the test magnitude is $F_{6,6,0.95} = 4.28$ derived from F-Table for homogeneity test. The zero hypothesis is rejected because of $T_{I,II} > F_{6,6,0.95}$ at the end of comparison of the homogeneity test. The Model II is a more proper model than the stochastic model I based on homogeneity.

5.2 Experiments of Triangulation Net-1

Two triangulation nets are used for the triangulation nets experiments. The first application is Izmir Triangulation Net. The net consists of 10 points, 42 observations of direction and 17 observations of distance (Figure 2).

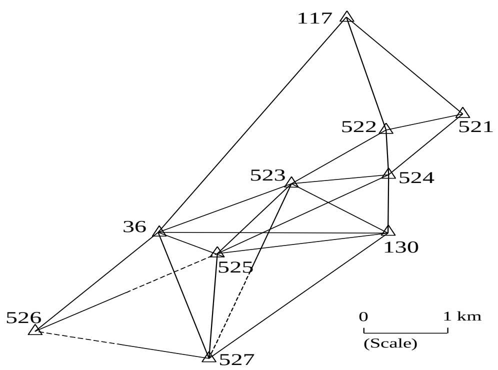

Figure 2: Triangulation net constituted for İzmir Metro

The triangulation net is adjusted by the least-square adjustment with free adjustment approach using both stochastic Model I and II. A-priori variances (before adjustment), a-posteriori variances (after adjustment) are given in Table 3.

Table 3: Stochastic Model Test ($F_{0.95,14,\infty} = 1.702$)

Stochastic Models	Before Adjustment	After Adjustment	Test Value	Zero Hypothesis
I	10.000	5.062	3.903	Not Accepted
II	1.000	1.024	1.040	Accepted

The differences, summations of long and short axis for homogeneity, and the mean accuracy tests are calculated by the differences and the summations, are given for both the stochastic Models I and II in Table 4.

Table 4: The Results of Triangulation Net Constituted for Izmir Metro

	Stochastic Models				
	I		II		
	$A_i + B_i$	$A_i - B_i$	$A_i + B_i$	$A_i - B_i$	Explanations
36	1.535	0.424	1.229	0.145	
117	2.845	0.803	2.211	0.428	
130	1.603	0.026	1.418	0.117	
527	2.242	0.714	1.418	0.327	
521	2.099	0.864	1.794	0.610	Sums and Differences of short and long axises of
522	1.372	0.524	1.720	0.267	Error ellipses of Points ($A_i + B_i$) ($A_i - B_i$)
523	1.246	0.289	1.098	0.142	
524	1.316	0.300	1.090	0.121	
525	1.414	0.318	1.199	0.049	
526	2.921	1.040	2.197	0.088	
Δ, η	1.859	0.530	1.508	0.229	Mean values of standard deviations
Δ, η	0.200	0.100	0.140	0.056	Standard deviations for homogeneity test and isotropy test
T_Δ, T_η	$T^{\Delta}_{I,II}$ = 2.041 , $T^{\eta}_{I,II}$ = 3.189				The test values for homogeneity test and isotropy test

The first criterion (stochastic model test), is applied to both the stochastic models. The stochastic model I does not accept the zero hypothesis and the stochastic model II accepts the zero hypothesis as seen in Table 3. In this case, it can be seen that the stochastic model II is a convenient model after stochastic model test.

An other criterion of examining the stochastic models is homogeneity. The test value for Stochastic models I and II $T^{\Delta}_{I,II} = 2.041$ is computed from equation 26 and the test magnitude $F_{9,9,0.95} = 3.180$ is from F-Table. The result of comparison of $T^{\Delta}_{I,II} \leq F_{6,6,0.95}$, it can be seen that the zero hypothesis is accepted. In another words, the stochastic model I and stochastic model II are equal to each other based on homogeneity. The mean accuracy test requires after the homogeneity test. The test value $t_{I,II} = 5.534$ is computed as in equation 22 and the test magnitude $t_{0.95,18} = 1.734$ is from the t-Table. The stochastic model II is superior than the stochastic model I after comparison of the test value and the test magnitude ($t_{I,II} > t_{0.95,18}$), based on mean accuracy of homogeneity.

The another test is the isotropy test for the results of adjustment which are calculated for both the stochastic models in the triangulation nets. The test value $T^{\eta}_{I,II} = 3.189$ is computed from equation 32 and the test magnitude $F_{6,6,0.95} = 3.180$ is from F-Table. It can be seen that the stochastic model II is superior than the stochastic model

I after comparison of the test value and the test magnitude ($T^{\eta}_{I,II} > F_{9,9,0.95}$), based on isotropy.

5.3 Experiments of Triangulation Net-2

The second net is a part of the Istanbul net. The net consists of 16 points, 68 observations of directions and 35 observations of distances (Figure 3).

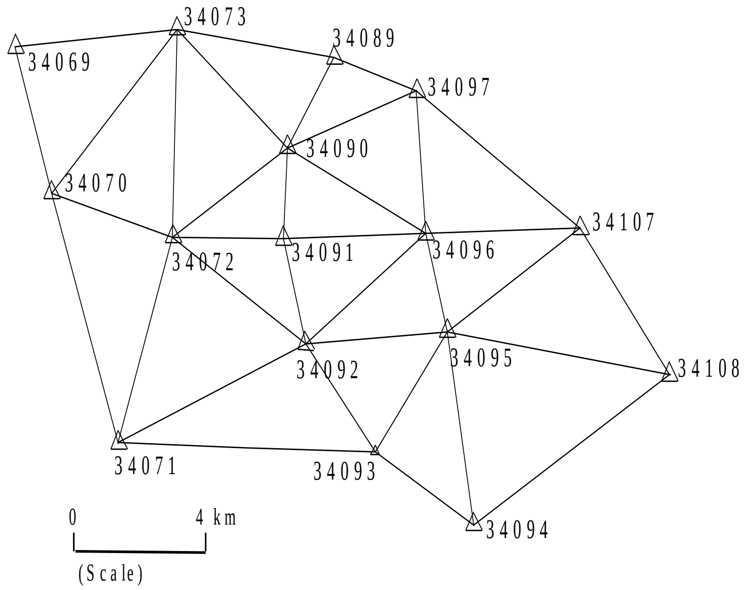

Figure 3: Triangulation Net Constituted for Istanbul

The second triangulation net is adjusted by the least-square method with free adjustment approach using both the stochastic model I and II. The summarized results are given in Table 5 according to the stochastic models I and II.

Table 5: Stochastic Model Test ($F_{0.95,32,\infty} = 1.446$)

Stochastic Models	Before Adjustment	After Adjustment	Test Value	Zero Hypothesis
I	1.000	2.326	5.410	Not Accepted
II	1.000	1.130	1.277	Accepted

The stochastic model test, which is the first criterion for examining, is applied to the stochastic models and as seen in Table 5, the stochastic model named as I does not accept the zero hypothesis and the other stochastic model named as II accepts the zero hypothesis. The other explanation of the above results is that the stochastic model I is not a convenient model and the stochastic model II is convenient model.

The sums and differences of long and short axis, homogeneity and mean accuracy tests calculated by the differences and the summations, are given for both models in Table 6. The second criterion which is the homogeneity test, is applied to the results of triangulation net adjustment calculated by both the stochastic models. The test value $T^{\Delta}_{I,II} = 217.858$ is computed from equation 26 and the test magnitude $F_{15,15,0.95} = 2.400$ is from F-Table. The zero hypothesis is rejected after the comparison of the test value and the test magnitude ($T^{\Delta}_{I,II} > F_{15,15,0.95}$). The stochastic model II is a more proper model than the stochastic model I based on homogeneity.

The last test for triangulation nets is the isotropy test. The test is applied to the results of both the stochastic models (Table 6).

The test value $T^{\eta}_{I,II} = 1.100$ is computed from equation 32 and the test magnitude $F_{15,15,0.95} = 2.400$ is from F-Table. Both of stochastic models I and II are equivalent each other after comparing the test value and the test magnitude ($T^{\eta}_{I,II} \le F_{15,15,0.95}$) based on isotropy. For this reason, the mean accuracy test requires for the isotropy. The test value $t_{I,II} = 1.060$ is computed as in equation 22 and the test magnitude $t_{0.95,30} = 1.697$ is from the t-Table. The test comparison shows ($t_{I,II} \le t_{0.95,30}$) that both of stochastic model are equivalent in terms of isotropy.

Table 6: The Results of Triangulation Net Constituted for Istanbul

	Stochastic Models I		Stochastic Models II		Explanations
	$A_i - B_i$	$A_i + B_i$	$A_i - B_i$	$A_i + B_i$	
34069	4.262	1.237	3.579	1.052	Sums and differences of short and long axises of error ellipses of points ($A_i - B_i$) ($A_i + B_i$)
34070	3.014	1.454	2.455	1.361	
34071	3.798	1.050	2.852	1.061	
34072	1.972	1.331	1.786	1.249	
34073	2.730	1.516	2.306	1.402	
34089	2.469	1.201	2.393	1.148	
34090	1.698	1.160	1.714	1.232	
34091	1.720	1.038	1.787	1.083	
34092	1.856	1.088	1.782	1.077	
34093	2.568	1.628	2.356	1.561	
34094	3.490	1.463	3.128	1.514	
34095	1.789	1.245	1.769	1.121	
34096	1.621	1.236	1.747	1.072	
34097	2.572	1.215	2.536	1.101	
34107	2.806	1.442	2.500	1.256	
34108	3.860	1.225	3.316	1.231	
Δ, η	2.639	1.283	2.375	1.220	Mean values of sums and differences
Δ, η	2.214	0.043	0.150	0.041	Standard deviations for homogeneity test and isotropy test
T_Δ, T_η	$T^{\Delta}_{I,II}$ = 217.858 , $T^{\eta}_{I,II}$ = 1.100				The test values for homogeneity test and isotropy test

6. Conclusion

A stochastic model problem of geodetic nets has been investigated using the variance component estimation of Helmert type. To carry out the objective stated above, a leveling net and two different triangulation nets have been used. In the leveling net, each leveling line has three different measurements, which are precise, reciprocal trigonometric and leap-frong trigonometric leveling. The triangulation nets have direction and distance observations. Two different stochastic models have been applied to the above nets. The first model is the conventional approach, defined as the square of unit variance dividing by square of observation variance. The second stochastic model is the variance component estimation of Helmert type. Three criteria for the leveling net and five criteria for the triangulation nets have been taken into consideration in order to test the stochastic models. The researches have indicated that the Helmert type gave better reliable results than the conventional approach in the leveling net and the first triangulation net in terms of all the criteria. However, in the second triangulation net, the Helmert type gave the similar results with the conventional approach in terms of isotropy, and it is better in terms of all the other criteria.

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