ARAŞTIRMA MAKALESİ

AUE (ALMOST UNBIASED ESTIMATION) METHOD FOR THE ESTIMATION OF VARIANCE COMPONENTS IN HORIZONTAL CONTROL NETWORKS

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YATAY KONTROL AĞLARINDA VARYANS BİLEŞENLERİNİN KESTİRİMİ İÇİN AUE METODU

ÖZET

Bu çalışmada yatay kontrol ağlarının dengelenmesinde gerçekçi bir biçimde oluşturulması gereken stokastik modelin belirlenmesi için geliştirilmiş olan varyans bileşen tahmin yöntemi AUE incelenmiştir. İncelenen yöntem yatay kontrol ağlarının dengelenmesinde halen yaygın olarak kullanılan klasik dengeleme modeli (stokastik model) ile karşılaştırılmıştır. Her iki modelin karşılaştırılmasına yönelik olarak uygulanabilecek istatistiksel testlerden yararlanarak somut karar verme kriterleri tanımlanmış ve modellerden hangisinin daha üstün olduğu belirlenmeye çalışılmıştır. Modellerin karşılaştırılması amacıyla İstanbul Metropolitan Nirengi Ağının bir bölümüne (Anadolu Yakası) ilişkin veriler kullanılarak sayısal uygulama yapılmıştır.

SUMMARY

In this study, AUE variance component estimation method, which has been developed to determine the stochastic model, which is necessary to be formed in a real way in the adjustment of horizontal control networks, has been researched. The method has been researched has been compared with conventional model (stochastic model), which is still used widely in the adjustment of horizontal control networks. In order to compare both models, concrete deciding criteria, using statistical tests, have been defined and the determination of which model is superior has been studied. For the aim of comparison of the models, numerical experiment using data, which belong to the part of Istanbul Metropolitan Triangulation Network (Asiatic side), has been performed.

1. INTRODUCTION

Variance estimation method has become a very important topic in the geodetic community in last thirty years, however variance estimation method hasn’t been taken into consideration sufficiently in Turkish geodetic community.

Variance components have to be estimated from the observation data for realistic weighting. Information, that which are available about the precision of the observations (standard deviations and weights) and the correlation among them, before adjustment (a priori), is called stochastic model. In the adjustment of geodetic networks which need high accuracy and precision, variance components (weights) has to be estimated using observation data. Stochastic models, which don’t implicate the precision of the observations and the correlation among them in a real way, cause model errors. These errors are very important stochastic error sources in a least square adjustment (Öztürk 1987). In geodetic projects, which need high precision, in order to determine precisely the exact values of the unknowns and the accuracy criteria, observations has to be weighted suitably using variances and covariance of the observations. In order to obtain reliable results in an adjusted net, variance and covariance of the observations have to be expressed with models which are suitable to real conditions like instruments models used for surveying, surveying times, meteorological conditions during surveying, persons who have performed surveying, observation distances (short or long distances and directions concerned with these distances). In another words, obtaining of true and reliable results after adjustment depend on determining of realistic observation weights. This is very important for making adjustment of observations and analysing of surveying results. As known, the weight is a coefficient which express the accuracy and the reliability of the observations (Öztürk 1987). Nowadays, some conventional stochastic models are used widely to determine the observation weights without enough examination. Determination of the observation weights mentioned above may be enough for geodetic and engineering projects, which don't need high precision, but for geodetic and engineering projects, which need high precision, this may be inadequate.

In order to estimate variance components (weights) from the observation data, more realistic techniques have been developed like Helmert Variance Component Estimation Method, MINQUE (Minimum Norm Quadratic Estimation) Method, AUE (Almost Unbiased Estimation) Method and the method given by Förstner for the estimation of variance components.

2. STOCHASTIC MODEL

In order to form stochastic model, which is one of the mathematical components of the adjustment, variance - covariance matrix, which has been formed depending on a priori or a posteriori informations, which give the variances of the observations and correlation between the observations, has to be known. Variance - covariance matrix is

$$C_l = \begin{vmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_1\sigma_2 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{n1}\sigma_1\sigma_n & \rho_{n2}\sigma_2\sigma_n & \cdots & \sigma_n^2 \end{vmatrix} \qquad (2.1)$$

where

σ_i^2 : variances of the observations (i , j = 1 , 2 , 3 , ..., n)

ρ_{ij} : correlation between the observations ($i \neq j$)

correlation coefficient, which is known as the criterion of stochastic dependence between the observations and taken the value between $-1 \leq r \leq +1$, is defined as

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \qquad (2.2)$$

Where $\sigma_{i,j}$ denotes unknown covariance components.

Because of the determination of numerical equivalents of algebraic and physical relations between observations is generally impossible, observations are accepted independent from each other then the correlation coefficients are taken as zero. So, variance - covariance matrix can be written as,

$$C_l = \begin{vmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma_n^2 \end{vmatrix} \qquad (2.3)$$

As a result of the multiplication of inverse of this matrix with a suitable a priori variance component (σ_0^2),

$$\underline{P}_l = \sigma_0^2 \underline{C}_l^{-1} = \underline{Q}_l^{-1} \qquad (2.4)$$

the weight matrix of the observations (stochastic model) is obtained.
Supposing that there is no correlation between the observations, weight and cofactor matrixes can be written in diagonal form,

$$\underline{P}_l = \text{diag}(P_1, P_2, \ldots\ldots, P_n)$$

$$\underline{Q}_l = \text{diag}(q_1, q_2, \ldots\ldots, q_n)$$

Let σ_0^2 be a priori value of the unit weighted observation. Then the weight of l_i observation is calculated as

$$P_i = (l/q_i) = \sigma_0^2/\sigma_i^2 \qquad (i = 1, 2, \ldots\ldots, n) \qquad (2.5)$$

2.1 Estimation Of Variance Components Using Conventional Stochastic Model

The standard deviations of a net directions, which are evaluated as groups, can be obtained either from Ferrero equation

$$\tilde{\sigma}_0 = \pm\sqrt{\frac{[ww]}{6n}} \qquad (2.6)$$

or from station adjustment

$$\tilde{\sigma}_d = \pm\sqrt{\frac{[vv]}{(n-1)(s-1)}} \qquad (2.7)$$

$$\tilde{\sigma}_0 = \pm\frac{\tilde{\sigma}_d}{\sqrt{n}} \qquad (2.8)$$

A priori variances of the distance observations can be obtained using either equation (2.9), which is given by the instrument manufacturer,

$$\tilde{\sigma}_s^{\,2} = a^2 + b^2 \times S^2 \qquad (2.9)$$

or equations (2.10) and (2.11)

$$\tilde{\sigma}_0 = \pm\sqrt{\frac{[gg]}{2n}} \qquad (2.10)$$

$$\tilde{\sigma}_s = \pm\frac{\tilde{\sigma}_0}{\sqrt{2}} \qquad (2.11)$$

where

g : observation difference
n : number of observations
$\tilde{\sigma}_0$: standard deviation of one of the observations, which are obtained by twice measuring of a side, whose weights are equal
$\tilde{\sigma}_s$: standard deviation of the average of the observations

After the computation of variances of distances and directions, assuming that a priori variance, determined for a group of directions, is a priori variance (σ_0^2) for unit weighted observation, and weights of the other direction groups and weights of distances in the net can be obtained from

weights of the unit weighted direction groups $P_i = 1$

weights of other direction groups $P_j = \frac{\sigma_0^2}{\sigma_j^2}$ (2.12) weights of distances $P_s = \frac{\sigma_0^2}{\sigma_s^2}$

Because of this model, which is used for the adjustment of horizontal control networks, doesn't reflect exactly the net geometry and the real conditions during surveying, adjustment results and their precision, determined by using this model, are questionable (Chen et al., 1990)

2.2 Estimation Of Variance Components Using AUE

This method has been developed by Horn et.al. in 1975 (Lucas, 1985). The method have to be applied in iterations. The brief of the method as follows

$$\underline{\sigma} = \underline{S}^{-1}\underline{q} \quad (2.13)$$

$$\underline{\sigma} = (\sigma_1^2, \sigma_2^2, \ldots\ldots, \sigma_m^2)^T$$

$$\underline{S} = Tr(W H_i W H_j) \qquad i, j = 1, 2, \ldots\ldots, m \quad (2.14)$$

$$\underline{q} = \underline{l}^T \underline{W} \underline{H}_i \underline{W} \underline{l} \qquad i = 1, 2, \ldots\ldots, m \quad (2.15)$$

These equations are equivalence with MINQUE (Minimum Norm Quadratic unbiased Estimation) equations (Lucas, 1985). At the point of convergence, variances of the observation groups

$$\underline{\sigma} = (1, 1, \ldots, 1)^T$$

then equation (2.13) becomes

$$\underline{\sigma}_i^2 = \frac{\underline{l}^T \underline{W} \underline{H}_i \underline{W} \underline{l}}{Tr(WH_i)} \quad (2.16)$$

Equation (2.16) is AUE equation for estimating the variance components concerning to the observation groups. Where,

$$\underline{W} = \underline{P} - \underline{P}\,\underline{A}\,\underline{N}^{-1}\,\underline{A}^T\underline{P} \quad (2.17)$$

$$H_i = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & P_i^{-1} & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

In this method, variance components are estimated by grouping the observations according to definite criterion. The method have to be applied in iterations (Lucas, 1985; Yavuz, 2000). A priori weights can be select as $P_1^{(0)} = P_2^{(0)} = \ldots\ldots = P_i^{(0)} = 1$ for all groups (Yavuz, 2000). After every iteration, the weights of the observation groups are computed using the following equation,

$$P_{i+1} = \frac{P_i}{\sigma} \tag{2.18}$$

When σ_i^2 =1 for all observation groups, iteration is finished .

On the iterative estimation of variance components, one or more of variance components can be negative during any iteration. Inexact grouping of observations can be the reason of this issue (Lucas, 1985). In the situation of getting negative estimations, the groups concerning to the observations must be rearranged. If observation groups won't be rearranged, instead of negative estimations, zero or very small positive number can be taken alternatively (Lucas, 1985).

3. GLOBAL TEST AND OUTLIER TEST

In order to test the compatibility of the estimated a posterior variance factor $\hat{\sigma}_0^2$ with a priori selected variance factor σ_0^2, the global test must be applied first. In this test, σ_0^2 is compared with $\hat{\sigma}_0^2$. Under the null hypothesis (H_0)

$$E\{\hat{\sigma}_0^2 | H_0\} = \sigma_0^2 \tag{3.1}$$

or

$$E\{(\hat{s}_0^2 / s_0^2) | H_0\} = 1 \tag{3.2}$$

Test statistic value

$$T = \hat{\sigma}_0^2 / \sigma_0^2 \quad , \quad \hat{\sigma}_0^2 > \sigma_0^2 \qquad \text{or}$$

(3.3)

$$T = \sigma_0^2 / \hat{\sigma}_0^2 \quad , \quad \sigma_0^2 > \hat{\sigma}_0^2$$

is compared with the value ($F_{f_1,f_2,1-\alpha}$ or $F_{f_2,f_1,1-\alpha}$) taken from F distribution table.

where

f_1 :degrees of freedom of $\hat{\sigma}_0^2$

f_2 :degrees of freedom of σ_0^2

$1-\alpha$:confidence limit

If

$$T > F_{f_1,f_2,1-\alpha} \quad , \hat{\sigma}_0^2 > \sigma_0^2 \qquad \text{or}$$

$$T > F_{f_2,f_1,1-\alpha} \quad , \sigma_0^2 > \hat{\sigma}_0^2 \qquad (3.4)$$

there is model error, in other words, weights of observations are not chosen properly and zero hypotheses is rejected,

If the global test failed and some residuals show excessive magnitude, outlier test (Baarda's data snooping or Pope tau-test) is employed

Test statistic values for these techniques mentioned above as follows

Baarda (data snooping) $$T_{i,B} = v_i / \sigma_0 \sqrt{q_{v_i v_i}} \sim N(0,1) \qquad (3.6)$$

Pope (tau-test) $$T_{i,P} = v_i / \hat{\sigma}_0 \sqrt{q_{v_i v_i}} \sim \tau_f \qquad (3.7)$$

where

$q_{v_i v_i}$: weight coefficient for residual v_i

If the test magnitudes exceed the limits below, it is accepted that outlier exist in the related observation data. In this case, this observation can be eliminated, and a new adjustment is made.

$$T_{max,B} > k_{1-\alpha_0/2} = \sqrt{F_{1,\infty,1-\alpha_0}} \qquad (3.8)$$

$$T_{max,P} > \tau_{f,1-\alpha_0/2} \qquad (3.9)$$

Where

$k_{1-\alpha_0/2}$: confidence limit taken from standard normal distribution table

$\tau_{f,1-\alpha_0/2}$: confidence limit taken from τ distribution table

After the tests been made, considering the condition that minimum number of the observations, which have outlier, has been eliminated during adjustment, the stochastic model which proves null hypothesis in eq. (3.2), is accepted superior than the other which reject the null hypothesis.

4. EXPERIMENTS

In this study, the part of Istanbul Metropolitan Triangulation Network (Asiatic side), which was surveyed in 1987, has been selected as application network. In order to compare the models taken in hand in this study, the net is adjusted by the least-square adjustment with free adjustment approach using both stochastic model. After testing the adjustment results, the stochastic models, selected for this study, has been compared using some comparison criterion, proposed by Yavuz [2000].

In this part, the conventional stochastic model in named as Model 1 and AUE method is named as Model 2. The measurements have to be divided into groups and the weights have to be determined iterativly in Model 2. Dividing the measurements into different numbered groups bring sub models into existence. This mentioned sub models have been indicated as
Stochastic Model No - the number of the measurement group
For example, the indication is like Model 2-2 for the stochastic Model 2 and two measurement group.

448 direction and 208 distance observations were made in the net. For Model 2, because of detailed information, which will form the base for grouping the measurements, like types of instruments used during surveying, which directions and distances were measured by using these instruments, date of measurements, has not been obtained, observations have been separated into two groups like a group made by using directions, another group made by using distances.

Observations have been also separated into four groups according to a distance criteria which is 5 kilometres.

In 5 km grouping criteria, number of observations in each group

-266 directions, which are shorter than the accepted distance criteria, form the first group.
-182 directions, which are longer than the accepted distance criteria, form the second group.
-123 distances, which are shorter than the accepted distance criteria, form the third group.
-85 distances, which are longer than the accepted distance criteria, form the fourth group.

In free net adjustment made by using Model 1, a priori standard deviation of unit weighted observation has been selected as $\sigma_0 = \pm 0.1136^{\text{mgon}}$ which was computed from station adjustment made for a station point. All computations have been made by using a personal computer which has the properties like pentium 133 CPU, 16MB Ram.

4.1 Comparison Of The Models According To Criteria 1

Needed time for the determination of the observation weights has been taken as criteria 1. It continued 9 minutes for Model1. For Model 2 which needs iteration

Model 2 -2 $3^h 22^m$

Model 2 -4 $6^h 45^m$

Iteration results concerning to the determination of the variance components (observation weights) have been given in Table 4.1 and Table 4.2.

Table 4.1: Results of the variance component estimation according to two observation groups for Model 2

MODEL NO 2				
ITERATION NO.	**1th.GROUP** σ_1^2	**2nd.GROUP** σ_2^2	**1th.GROUP WEIGHT**	**2nd.GROUP WEIGHT**
0			1	1
1	4.1002	5.3576	0.2439	0.1867
2	0.9640	1.0632	0.2530	0.1756
.	.	.	.	.
.	.	.	.	.
10	1.0000	1.0000	0.2574	0.1696

Table 4.2: Results of the variance component estimation according to four observation groups for Model 2

MODEL NO 2								
ITER NO	**1th.GR** σ_1^2	**2nd .GR** σ_2^2	**3rd.GR** σ_3^2	**4th.GR** σ_4^2	**1th.GR WEIGHT**	**2nd.GR WEIGHT**	**3rd.GR WEIGHT**	**4th.GR WEIGHT**
0					1	1	1	1
1	4.2518	3.9135	4.5490	6.5445	0.2352	0.2555	0.2198	0.1528
2	0.9902	0.9577	1.0287	1.0478	0.2375	0.2668	0.2137	0.1458
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
12	1.0000	1.0000	1.0000	1.0000	0.2388	0.2730	0.2050	0.1448

The final adjustment computation step has been finished in 14 minutes for both models. As a result, considering the total computation times, the superiority arrangement of the models according to criteria 1 can be given as

1^0 – Model 1 23^m

2^0 – Model 2 -2 $33^h 54^m$

3^0 – Model 2 -4 $81^h 14^m$

4.2 Comparison Of The Models According To Criteria 2

Global test (stochastic model test) on the a posterior variance factor has been applied to the adjustment results, which has been obtained by the adjustment of Asiatic Site using all measurements (448 directions and 208 distances), and the following results have been obtained. (comparison value taken from F distribution table

$F_{431,\infty,0.95} = 1.172$);

For Model 1 $T = \frac{1.848^2}{1.136^2} = 2.646$ > 1.172 null hypothesis is rejected

For Model 2 $T = \frac{1.085^2}{1^2} = 1.177$ > 1.172 null hypothesis is rejected

Because of null hypothesis is rejected for both models, gross error detection (localisation and elimination) tests have been applied and following results have been obtained (Table 4.3).

Table 4.3 : Gross error detection test results obtained by the free adjustment of Asiatic Site using all observations

Gross Error Detection Test		
	Direction observations with gross error	Distance observations with gross error
Model 1 Baarda-Data Snooping	22 directions	6 distances
Model 2-2, 2-4 Pope Tau and Baarda-Data Snooping	34138-34105 34138-34098 34098-34138	34069-34070 34106-34117

After the elimination of five observations with gross error (Table4.3) from the observations heap, the adjustment has been remade using both of the models and the global test (stochastic model test) on the a posterior variance factor has been reapplied to the adjustment results. (comparison value taken from F distribution table $F_{426,\infty,0.95} = 1.173$);

For Model 1 $T = \frac{1.568^2}{1.136^2} = 2.460$ > 1.173 null hypothesis is rejected

For Model 2-2 $T = \frac{1^2}{0.941^2} = 1.129$ < 1.173 null hypothesis is accepted

For Model 2-4 $T = \frac{1^2}{0.946^2} = 1.117$ < 1.173 null hypothesis is accepted

The results mentioned above prove that a lot of measurements must be eliminated from the observations heap for Model 1. But, because of the elimination of 28 observations with gross error (Table 4.3) might be weaken the net geometry, this has not been done. As a result, the superiority arrangement of the models according to criteria 2 can be given as

1^0 – Model 2-2, 2-4

3^0 – Model 1

4.3 Comparison Of The Models According To Criteria 3

Homogeneity test (criteria 3) (for further information look yavuz, [2000]) results have been given in Table 4.4.

Table 4.4: Homogeneity and mean accuracy test results applied to the adjustment results obtained by adjusting Asiatic Site net

$F_{75,75,0.95} = 1.4656$ $F_{150,0.95} = 1.6551$

MODEL NO	1 $\sigma^{(0)} = 0.0939$	2-2 $\sigma^{(0)} = 0.0689$	2-4 $\sigma^{(0)} = 0.0694$	SUPERIOR MODEL ACCORDING TO HOMOGENEITY	MEAN ACCURACY TEST VALUE	SUPERIOR MODEL ACCORDING TO MEAN ACCURACY
1		T=1.8573		2-2		
1			T=1.8307	2-4		
2-2			T=1.0146	equivalent	t=0.0567	equivalent

Because of Model 2-2 and 2-4 are equivalent and they are superior than Model 1 (Table 4.4) superiority arrangement of the models according to criteria 3 can be given as

1^0 – Model 2-2, 2-4

3^0 – Model 1

4.4 Comparison Of The Models According To Criteria 4

Isotropy test (criteria 4) (for further information look Yavuz, [2000]) results have been given in Table 4.5.

Table 4.5: Homogeneity and mean accuracy test results applied to the adjustment results obtained by adjusting Asiatic Site net

$F_{75,75,0.95} = 1.4656$ $F_{150,0.95} = 1.6551$

MODEL NO	1 $\bar{\sigma} = 0.0184$	2-2 $\bar{\sigma} = 0.0156$	2-4 $\bar{\sigma} = 0.0148$	SUPERIOR MODEL ACCORDING TO ISOTROPY	MEAN ACCURACY TEST VALUE	SUPERIOR MODEL ACCORDING TO MEAN ACCURACY
1		T=1.3912		equivalent	t=1.5298	equivalent
1			T=1.5457	2-4		
2-2			T=1.1110	equivalent	t=0.4516	equivalent

As a result, the superiority arrangement of the models according to criteria 4 can be given as

1^0 – Model 2-4

2^0 – Model 2-2

3^0 – Model 1

4.5 The evaluation of the results related to the adjustment of Asiatic Site net considering the whole criterion

The general superiority arrangement of the models, according to the whole criterion in the adjustment of Asiatic Site net, are summarised.

MODEL NO	CRITERIA 1	CRITERIA 2	CRITERIA 3	CRITERIA 4	TOTAL	GENEL ARRANGMENT
1	1	3	3	3	10	3
2-2	2	1	1	2	6	1
2-4	3	1	1	1	6	1

5. CONCLUSION

The stochastic models given by the methods like AUE, HELMERT, MINQUE etc. has to be used instead of the conventional stochastic model especially in the adjustment of geodetic nets which require higher accuracy and precision. These methods, besides conventional variance component estimation method, are already widely used in developed countries. Beside theoretical manner, the importance of these models have not been taken into consideration in Turkey until now. These models, which have been proved the superiorities against the conventional model by the aid of theoretical researches and practical applications as been done in this study, has to be used especially in the adjustment of geodetic nets, which require higher accuracy and precision, in private and public sector applications

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